Thermal and transport properties of cold dense matter inside neutrons stars.

- Transient phenomena in neutron stars
- Properties of solid and superfluid matter
- Transport properties of the core
- Conclusions

Collaborators:
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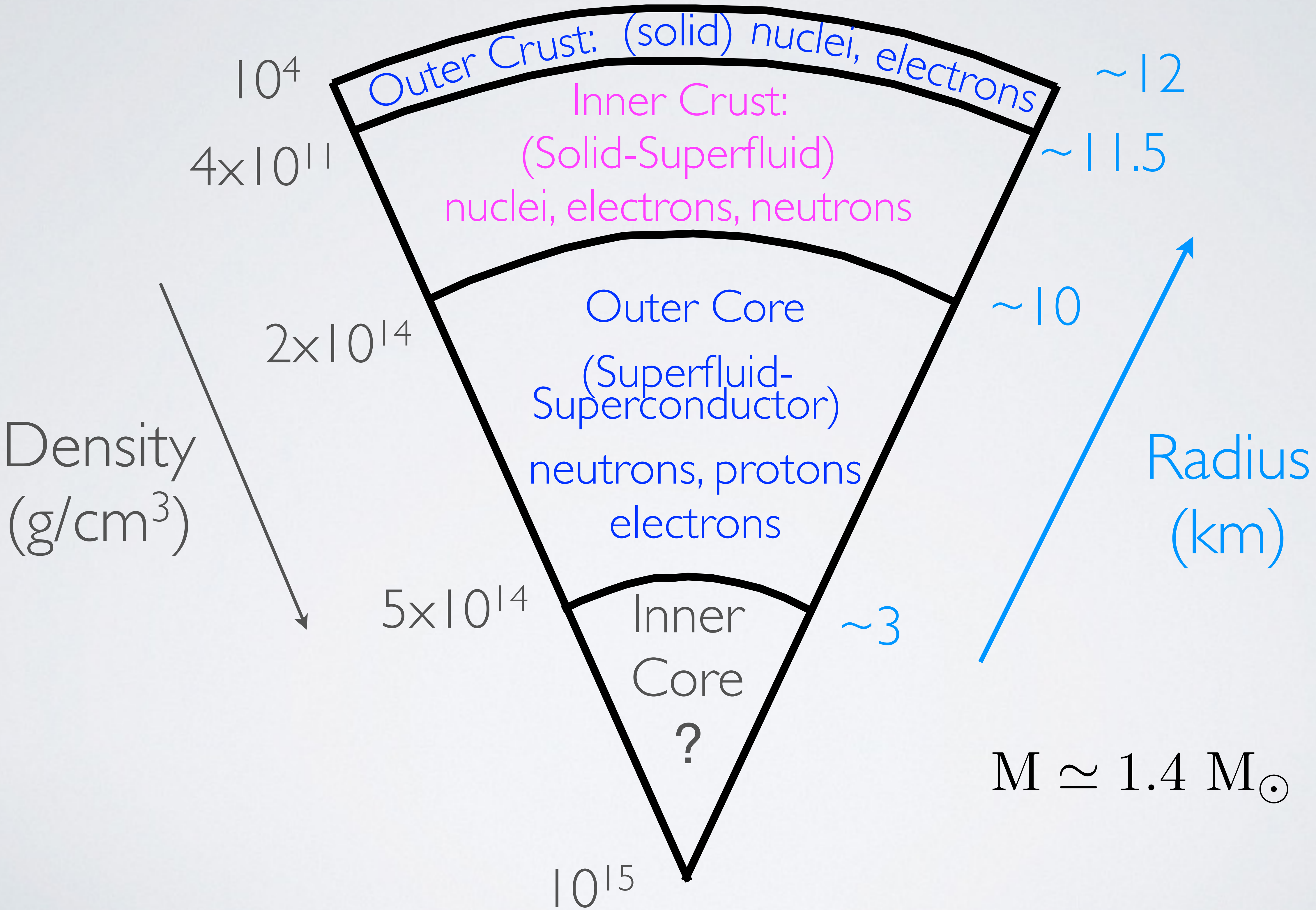
Relevant Papers:
arXiv:1409.7750
arXiv:1307.4455
arXiv:1210.5169
arXiv:1201.5602
arXiv:1102.5379
arXiv:1009.2303

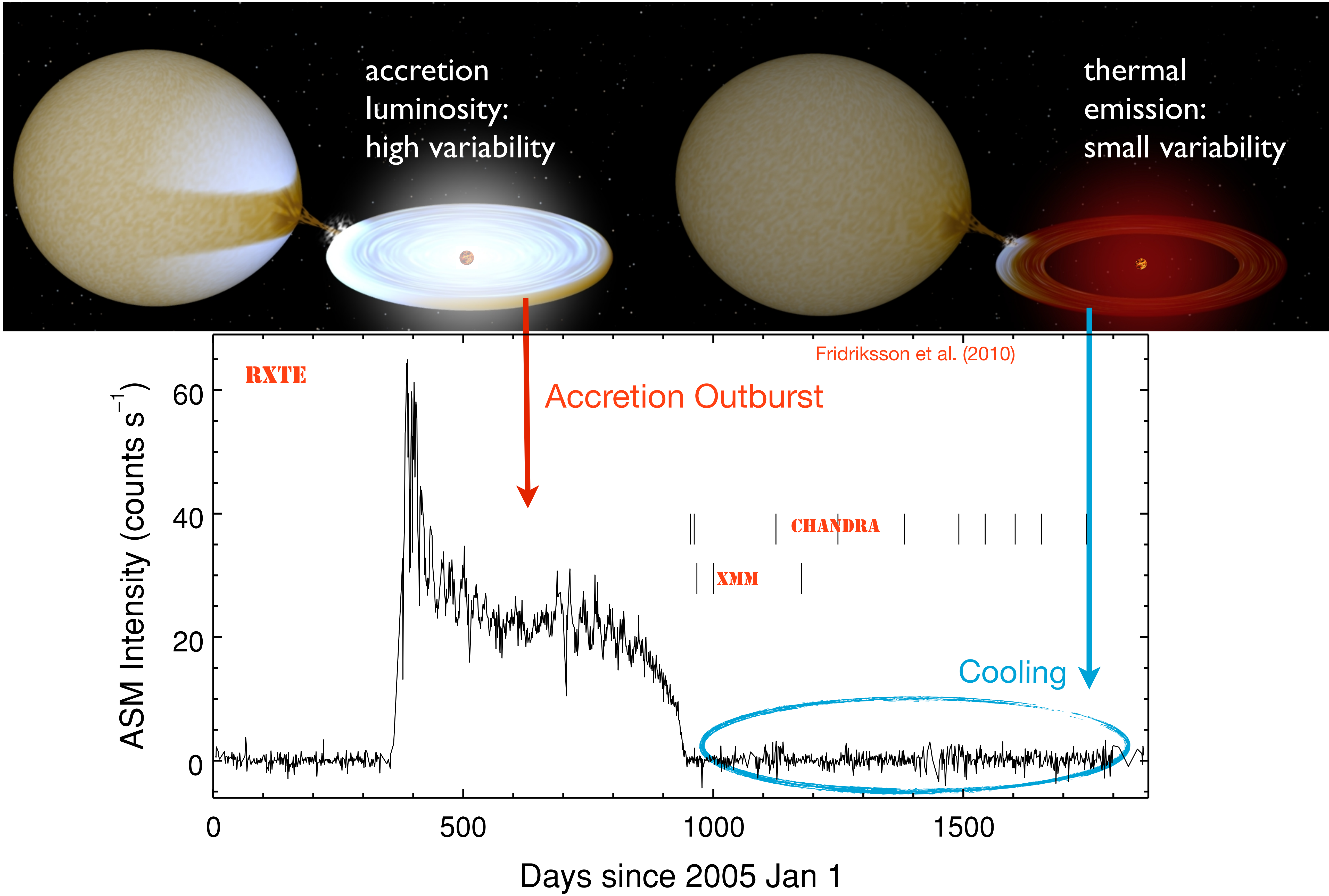
Phases of Dense Matter in Neutron Stars



Transient Phenomena in Neutron Stars

- Crustal heating and subsequent thermal relaxation in accreting neutron stars.
- Excitation of shear modes during magnetar giant flares.

States of an Accreting Neutron Star



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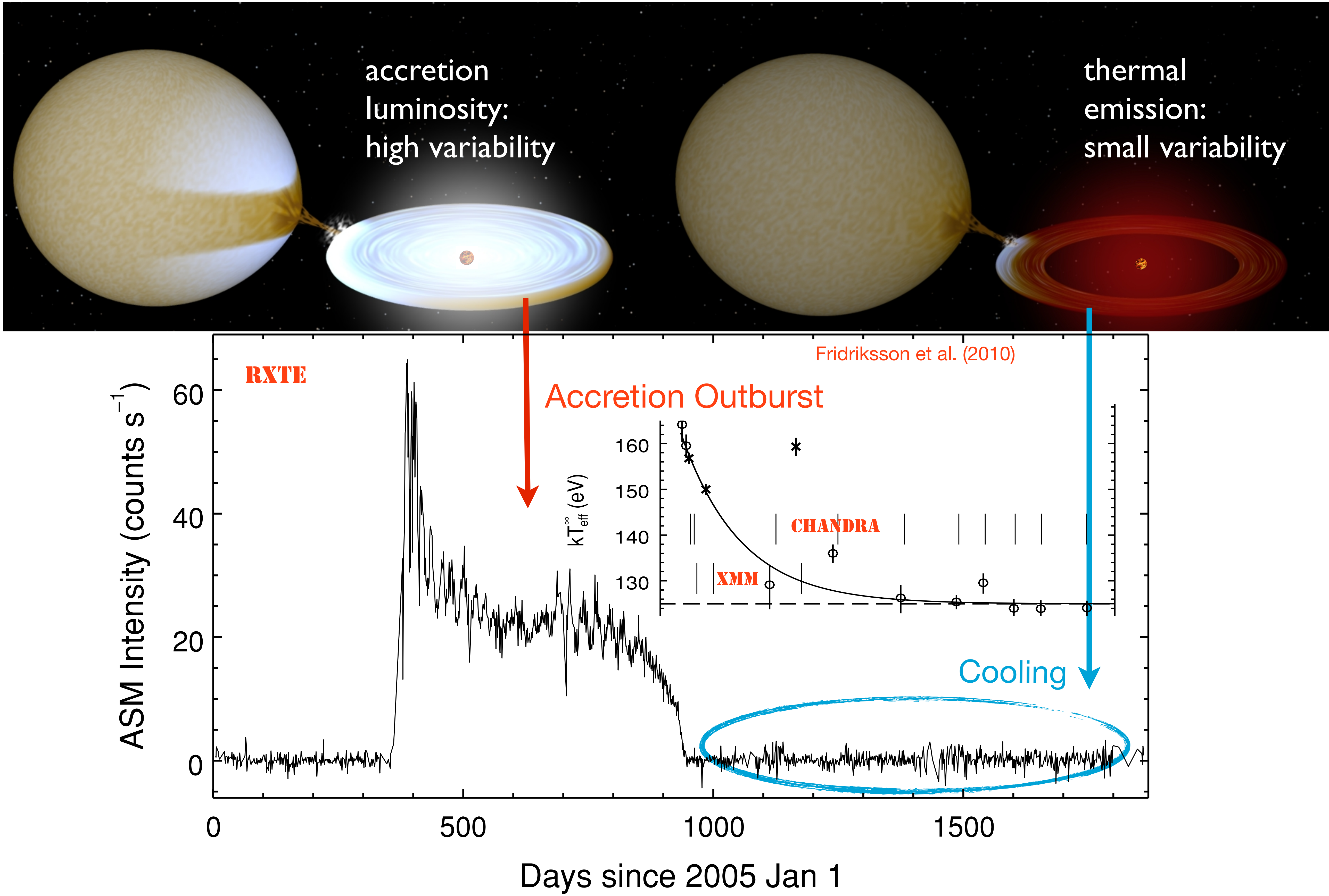
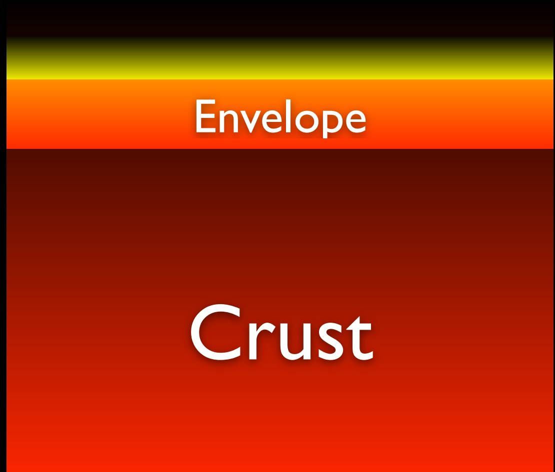


Transiently Accreting NSs

SXRTs: High accretion followed by periods of quiescence



Nuclear reactions release: ~ 1.5 MeV / nucleon

Deep crustal heating.

Brown, Bildsten Rutledge (1998)
Sato (1974), Haensel & Zdunik (1990)

Warms up old neutron stars

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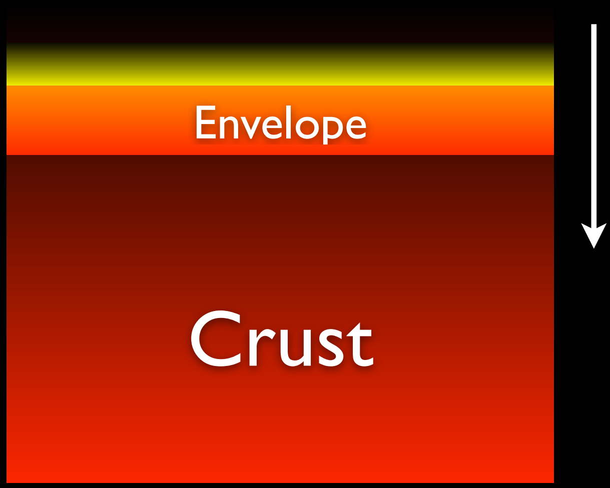

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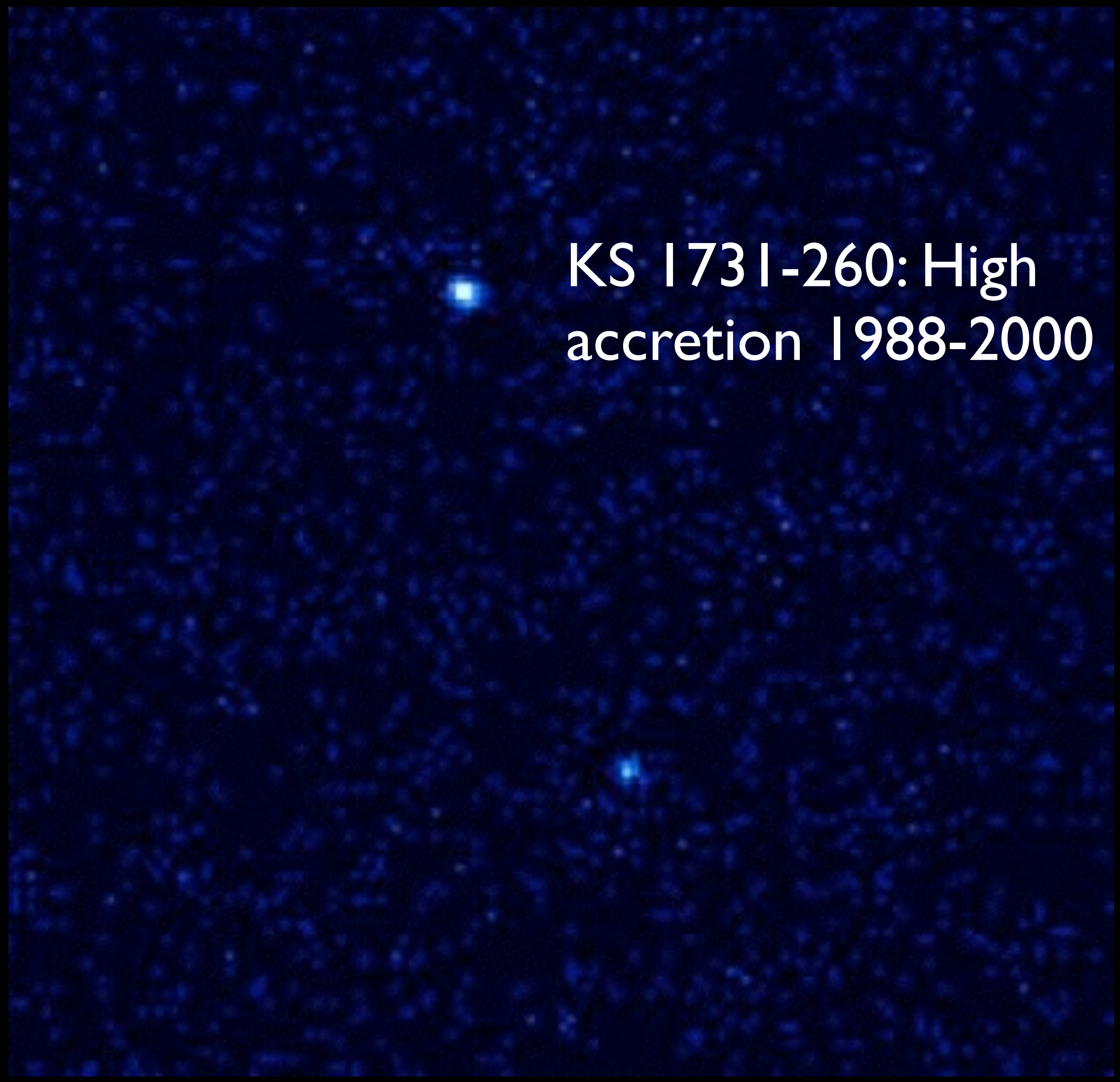
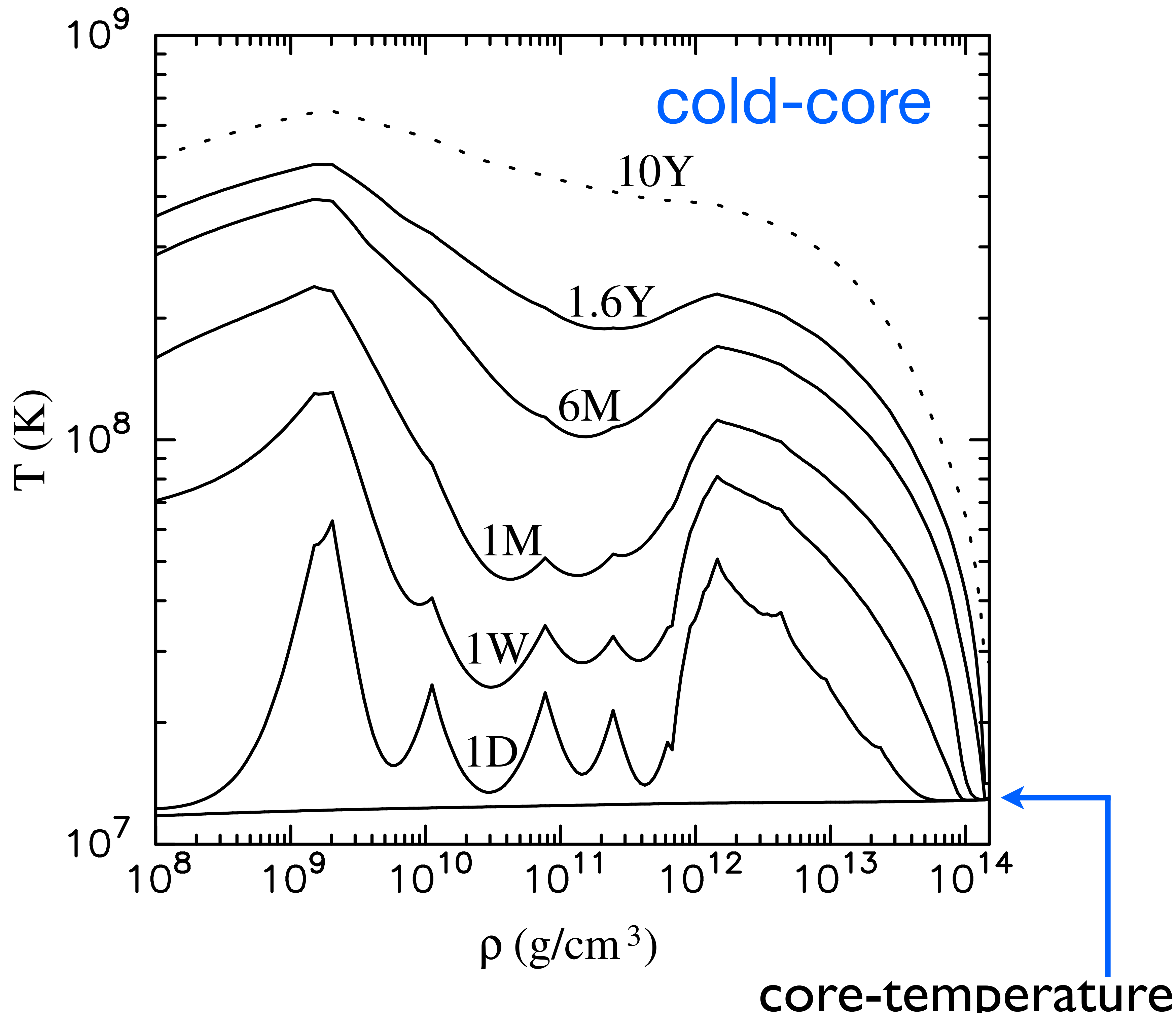


Image credit: NASA/CXC/Wijnands et al.

Accretion Induced Heating

Temperature profile depends on:

- accretion rate and duration.
- location of heat sources.
- thermal conductivity
- specific heat.
- core temperature



Page & Reddy (2012,2013)

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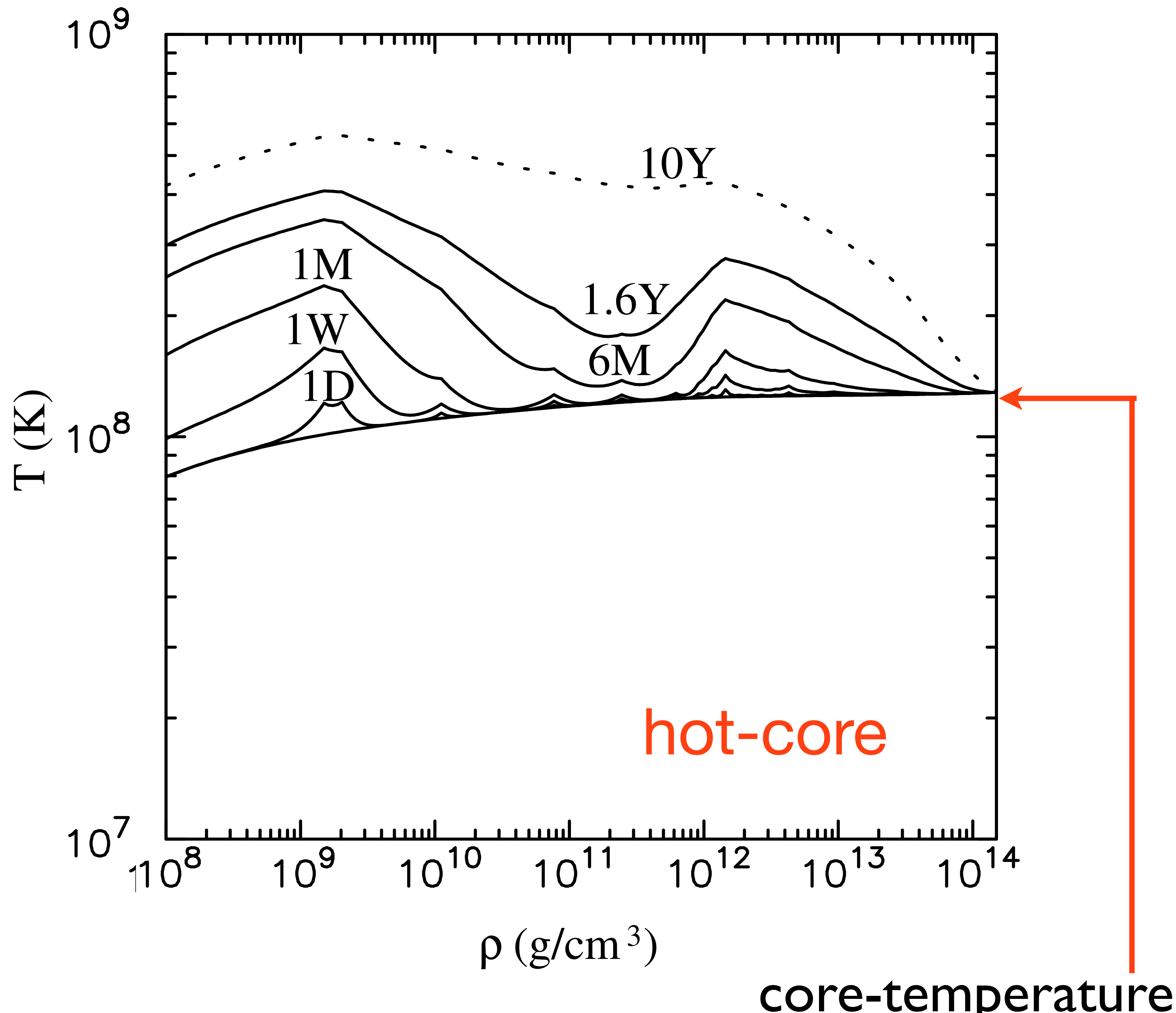
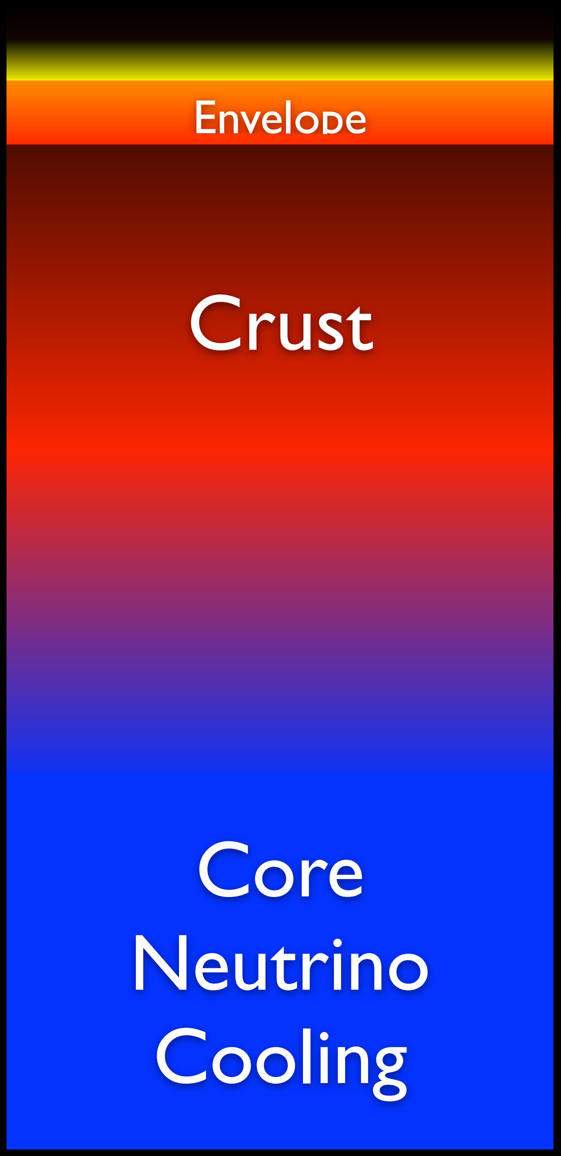


Page & Reddy (2012,2013)

Crust Cooling

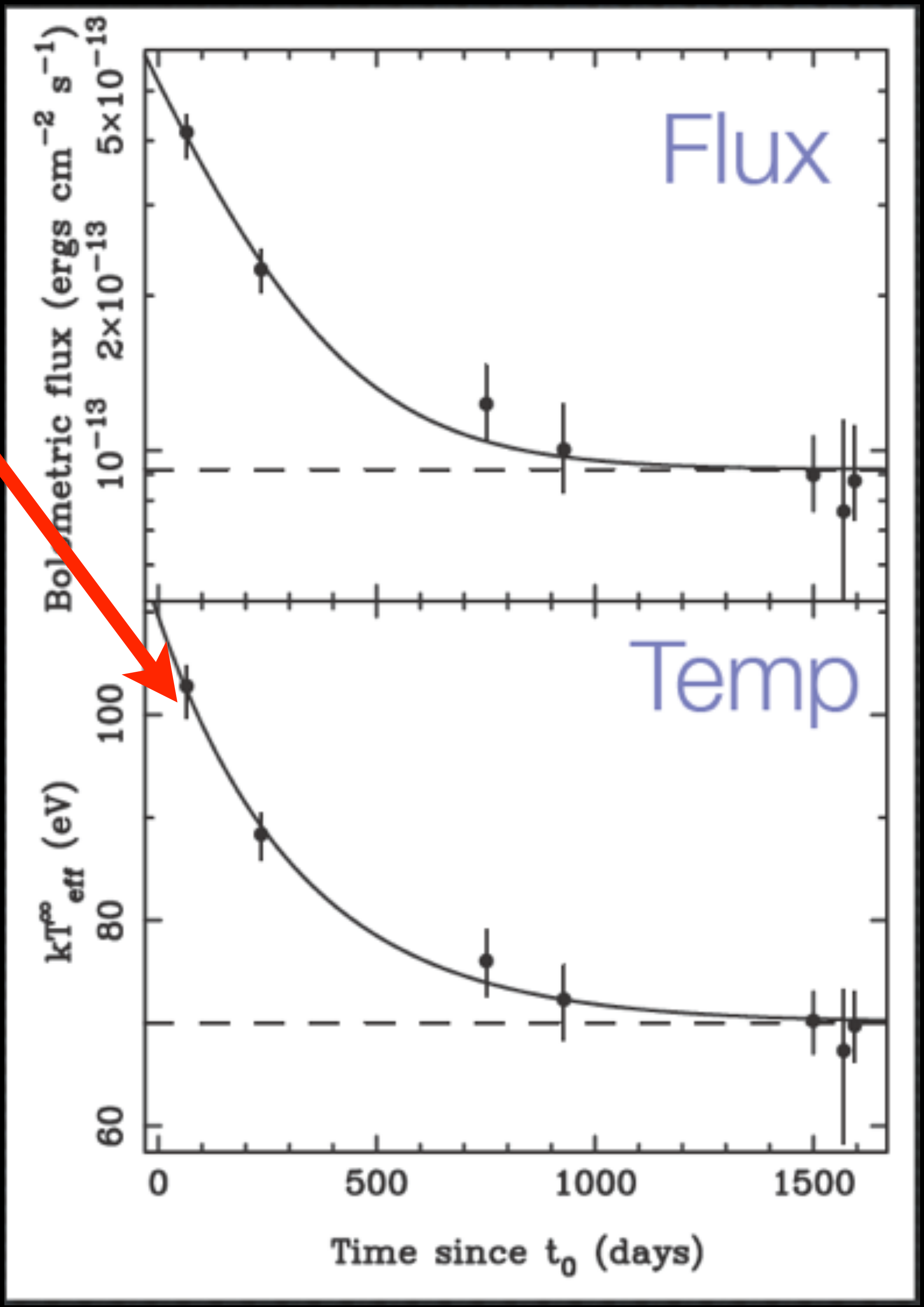
Watching NSs immediately after accretion ceases !



Crust Relaxation:

1. Initial temperature profile.
2. Thermal conductivity.
3. Heat capacity.

Shternin & Yakovlev (2007)
Cumming & Brown (2009)



Cackett, et al. (2006)

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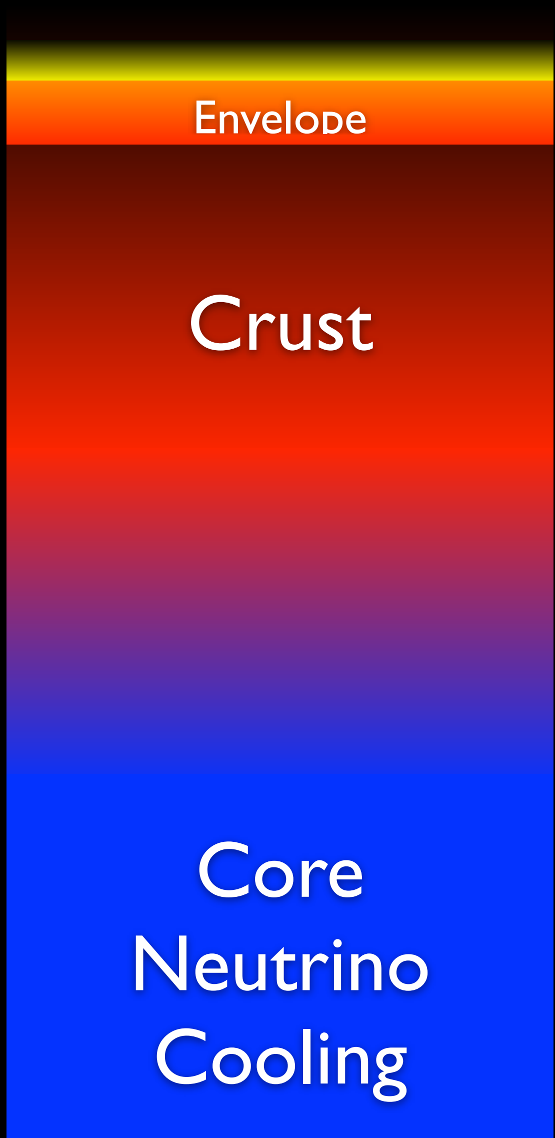

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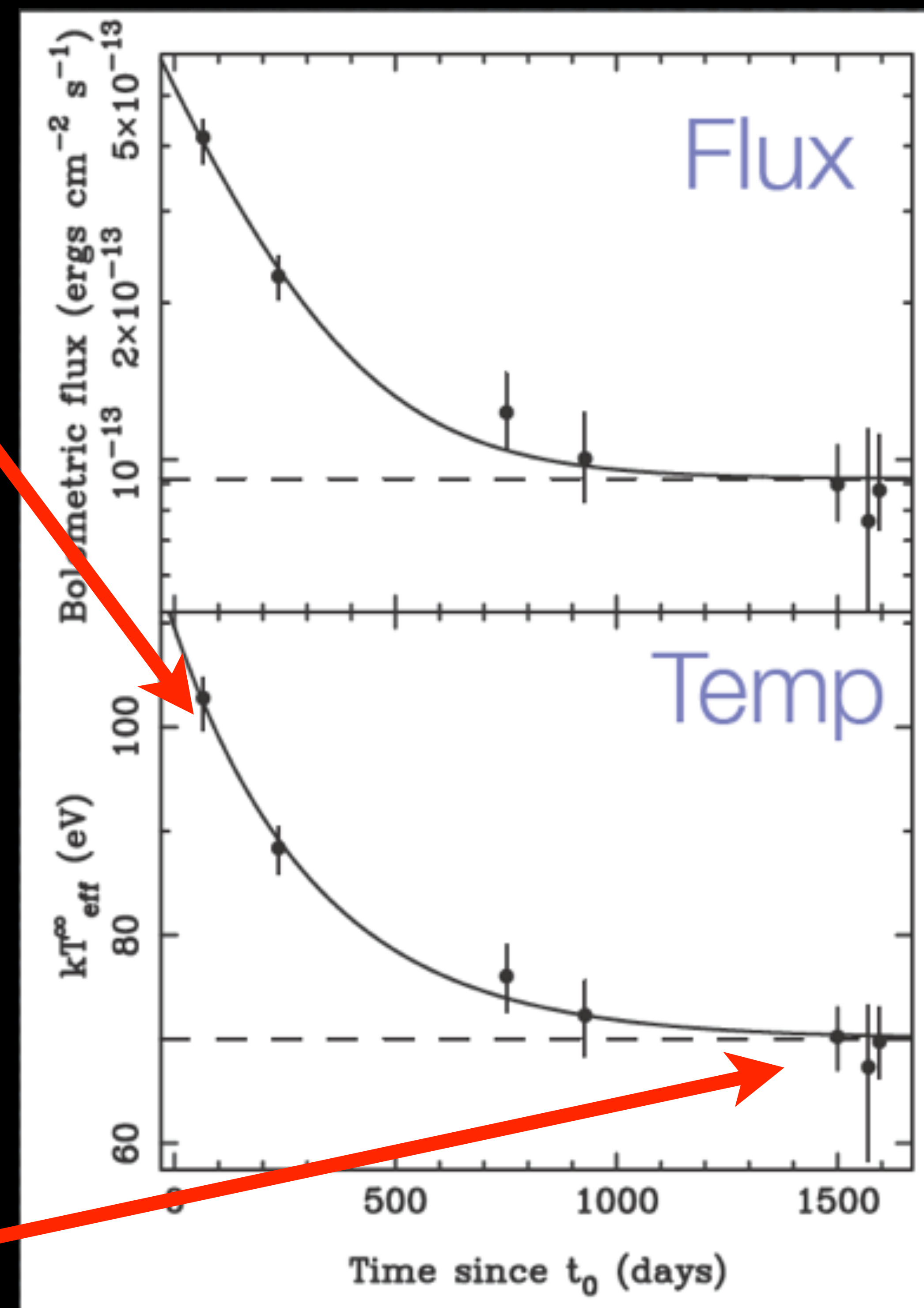


During quiescence we see the “Core Temperature”

Cackett, et al. (2006)

Observations:

All known Quasi-persistent sources with post outburst cooling

- After a period of intense accretion the neutron star surface cools on a time scale of years.

- This relaxation was first discovered in 2001 and 6 sources have been studied to date.

- Expected rate of detecting new sources ~ 1/year.

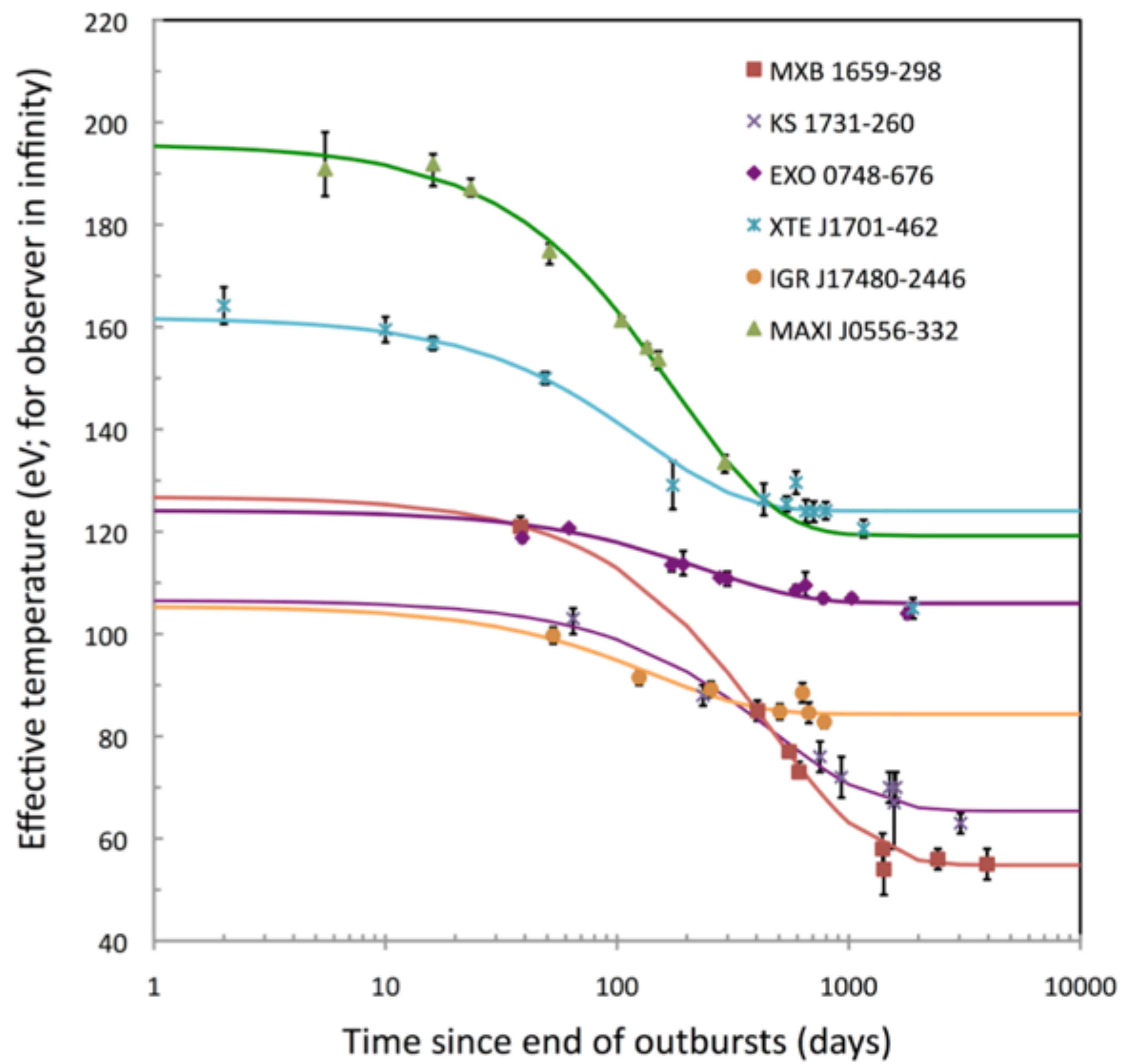


Figure from Rudy Wijnands (2013)

Connecting to Crust Microphysics

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa} \left(\Delta R\right)^2$$

Connecting to Crust Microphysics

Crustal Specific Heat

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa}\,(\Delta R)^2$$

Connecting to Crust Microphysics

Crustal Specific Heat

$$\tau_{\rm Cool} \simeq \frac{C_V}{\kappa} \; (\Delta R)^2$$

Thermal Conductivity

Connecting to Crust Microphysics

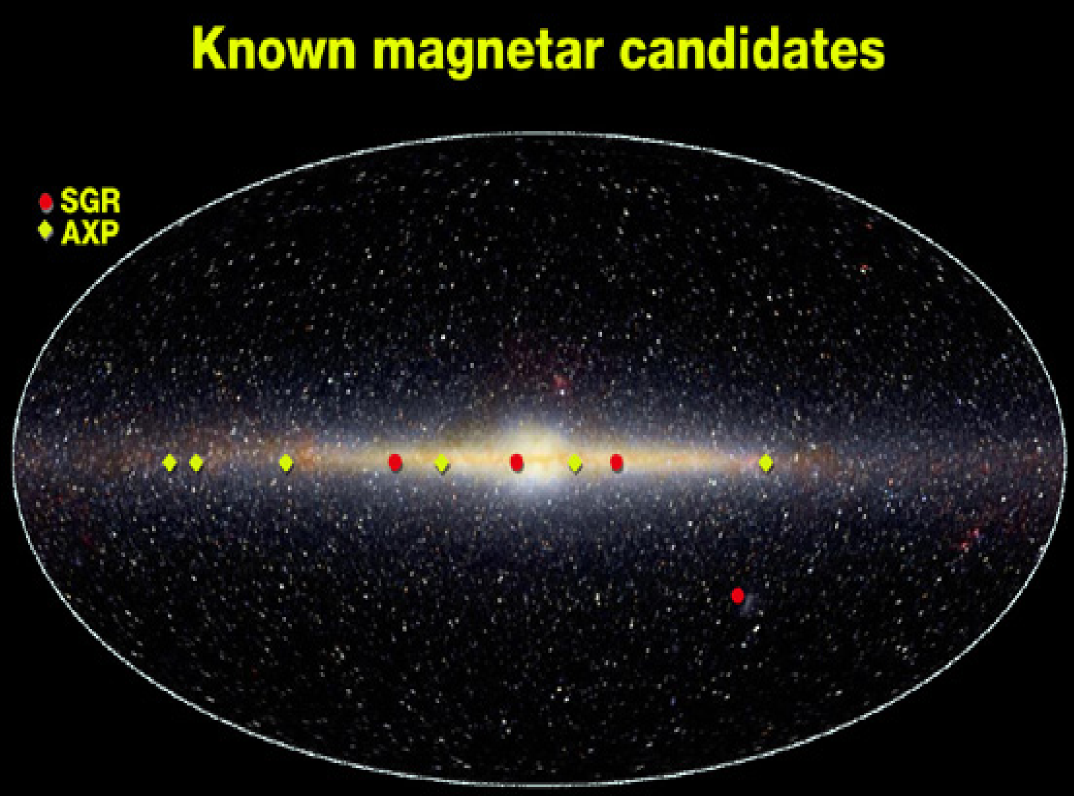
Crustal Specific Heat

Crust Thickness

$$\tau_{\mathrm{Cool}} \simeq \frac{C_V}{\kappa} (\Delta R)^2$$

Thermal Conductivity

Explosions on Magnetars: Giant Flares



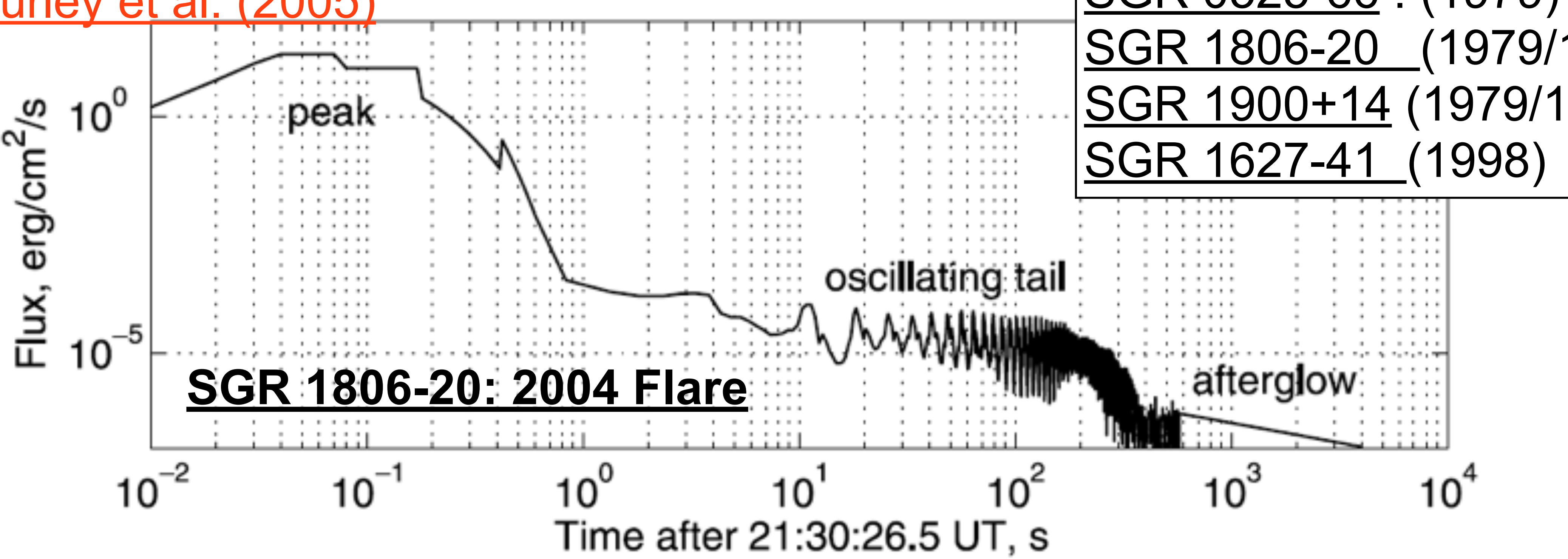
Anomalous X-Ray Pulsars (10)
Soft Gamma Repeaters (8)

Inferred to have surface fields of the order of 10^{15} Gauss.

http://www.physics.mcgill.ca/~pulsar/magnetar/main.html

SGRs exhibit powerful outburst ~ 10^{46} ergs/s

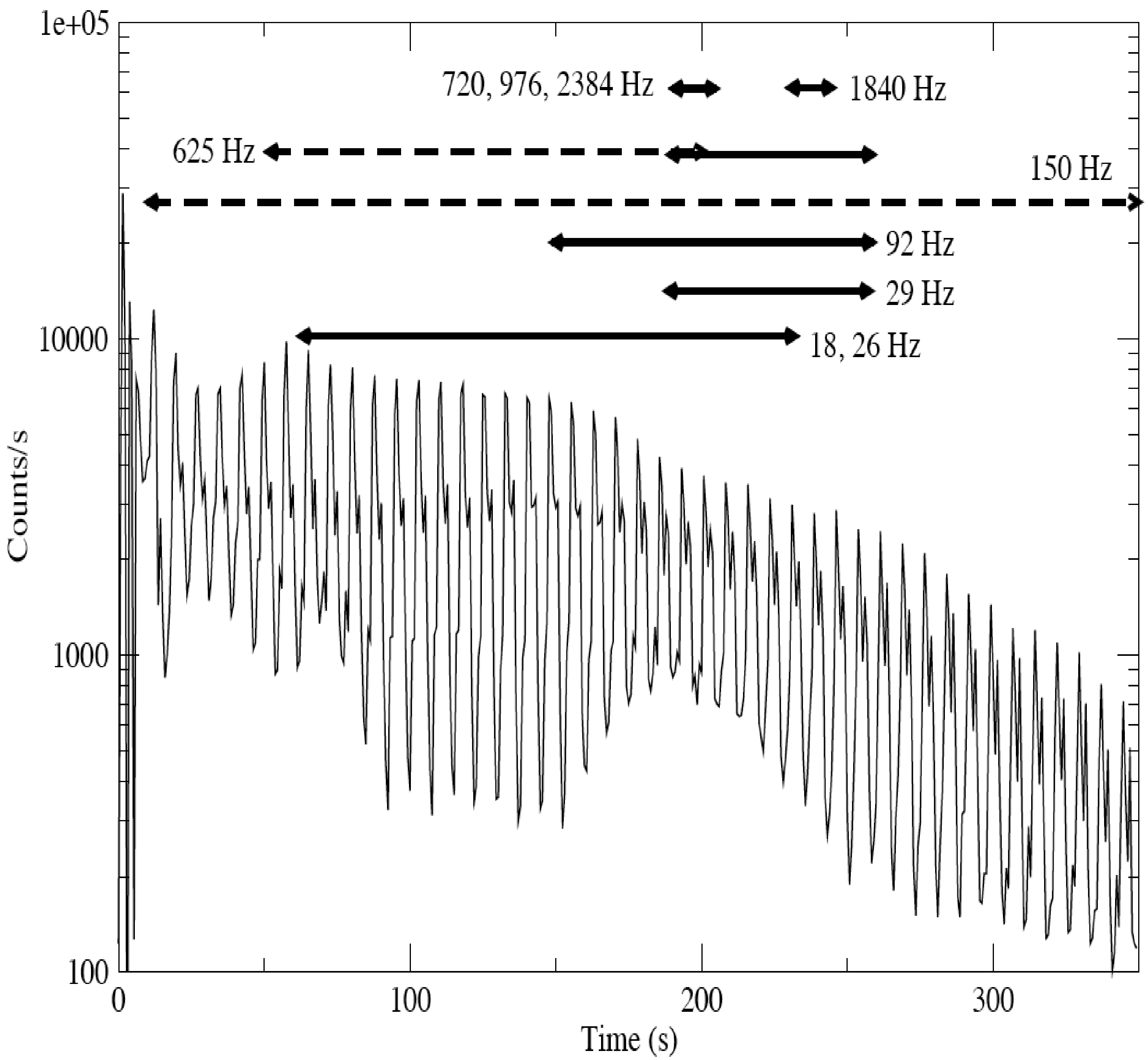
Hurley et al. (2005)



SGR 0525-66 : (1979)
SGR 1806-20 (1979/1986/2004)*
SGR 1900+14 (1979/1986/1998)
SGR 1627-41 (1998)

QPOs are likely to be shear modes in the solid crust

Duncan (1998), Strohmayer, Watts (2006)



SGR 1806
2004 Giant Flare

$$\omega_{\mathrm{n}=1} \simeq \frac{\pi v_t}{R} \frac{\Delta R}{R}$$

$$\omega_{\mathrm{n}=0,\mathrm{l}=2} \simeq \frac{2 v_t}{R}$$

Similar frequencies observed in 2 sources.

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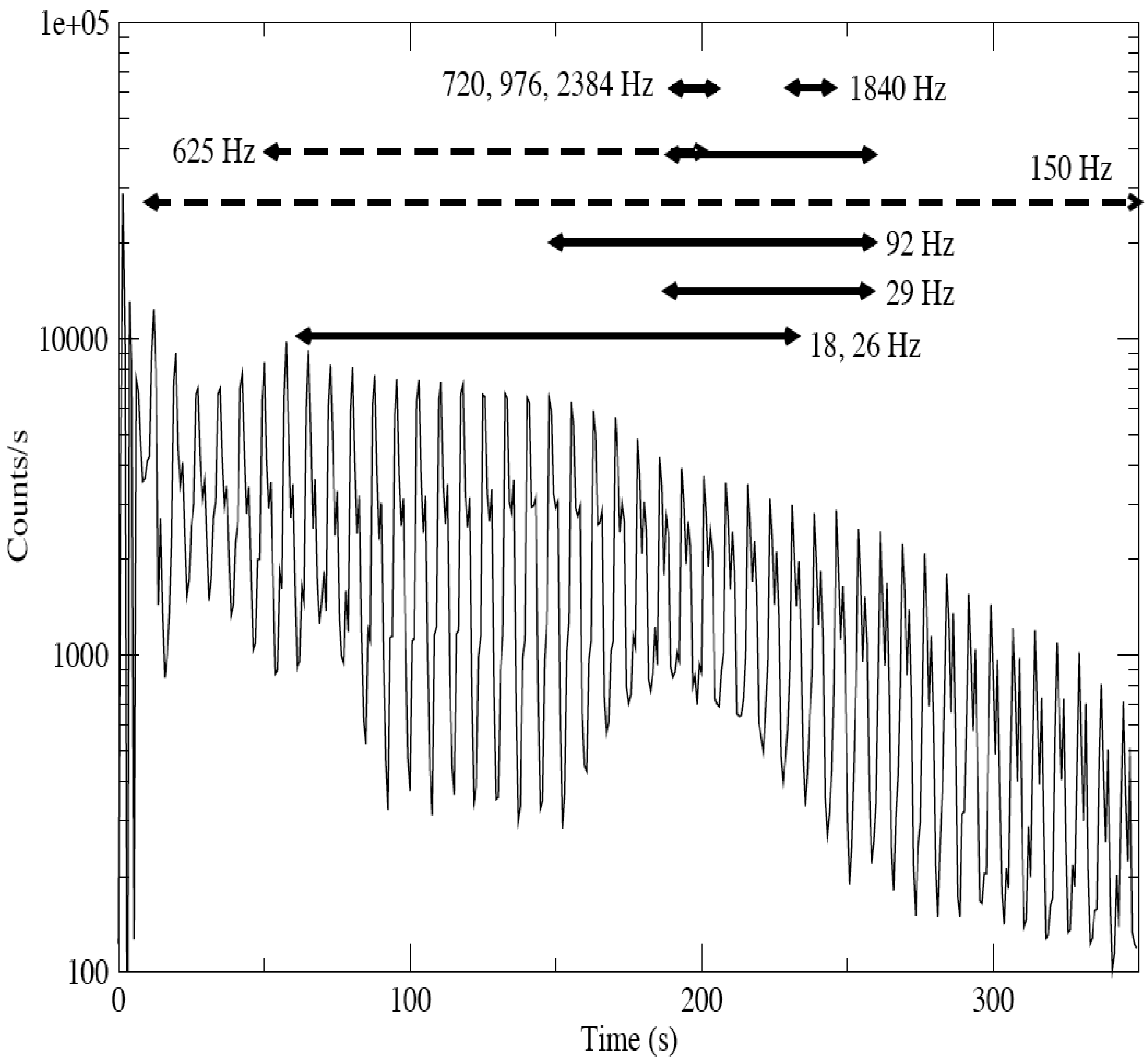

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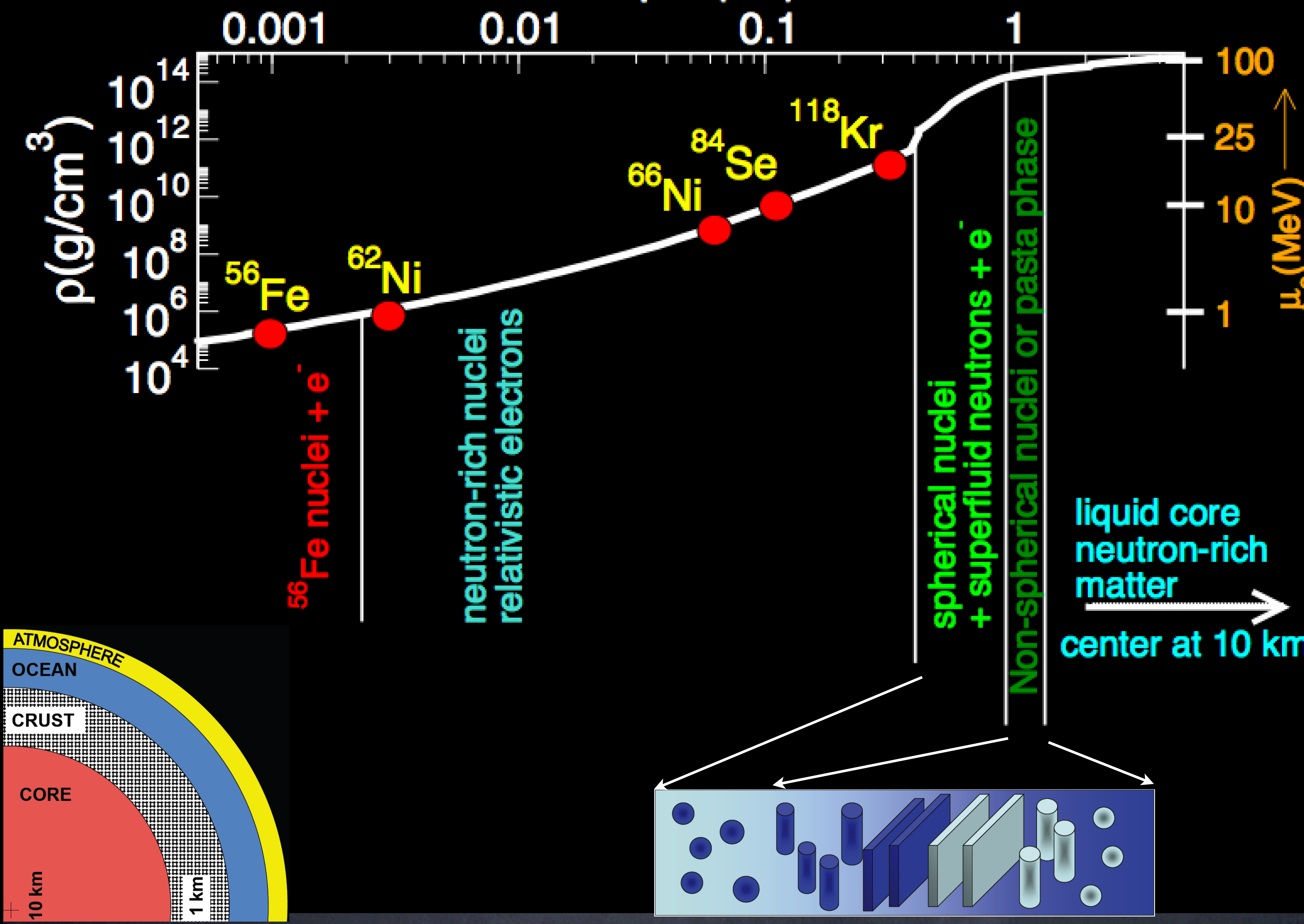
Shear mode velocity

Similar frequencies observed in 2 sources.

Neutron Star Crust:



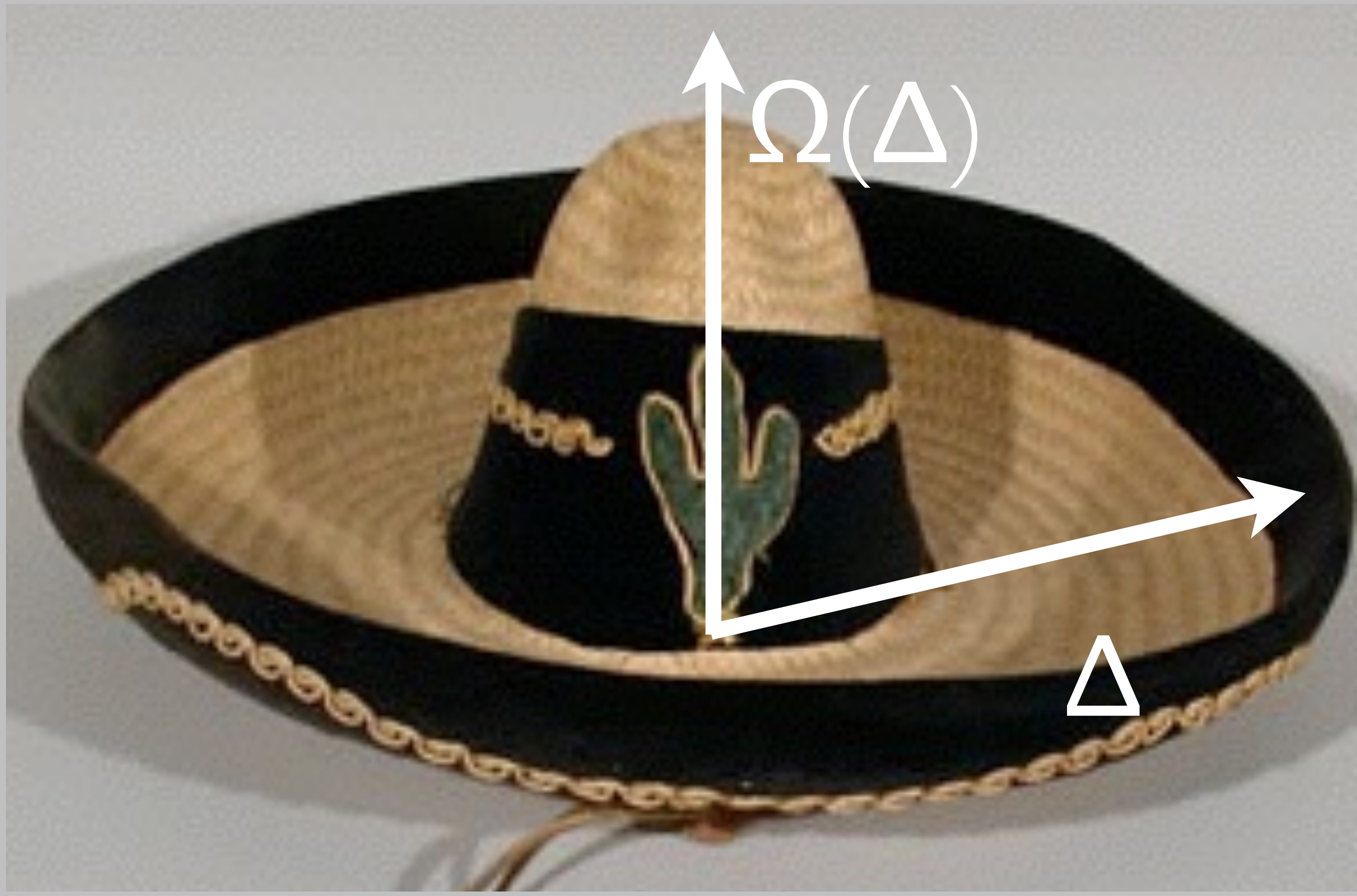
Cooper Pairing

Attractive interactions destabilize the Fermi surface:

$$H = \sum_{k,s=\uparrow,\downarrow} (\frac{k^2}{2m} - \mu)\ a^\dagger_{k,s} a_{k,s} + g \sum_{k,p,q,s=\uparrow,\downarrow} a^\dagger_{k+q,s} a^\dagger_{p-q,s} a_{k,s} a_{p,s}$$

$$\Delta = g\langle a_{k,\uparrow} a_{p,\downarrow}\rangle \quad \Delta^* = g\langle a^\dagger_{k,\uparrow} a^\dagger_{p,\downarrow}\rangle$$

Cooper pairs leads to superfluidity



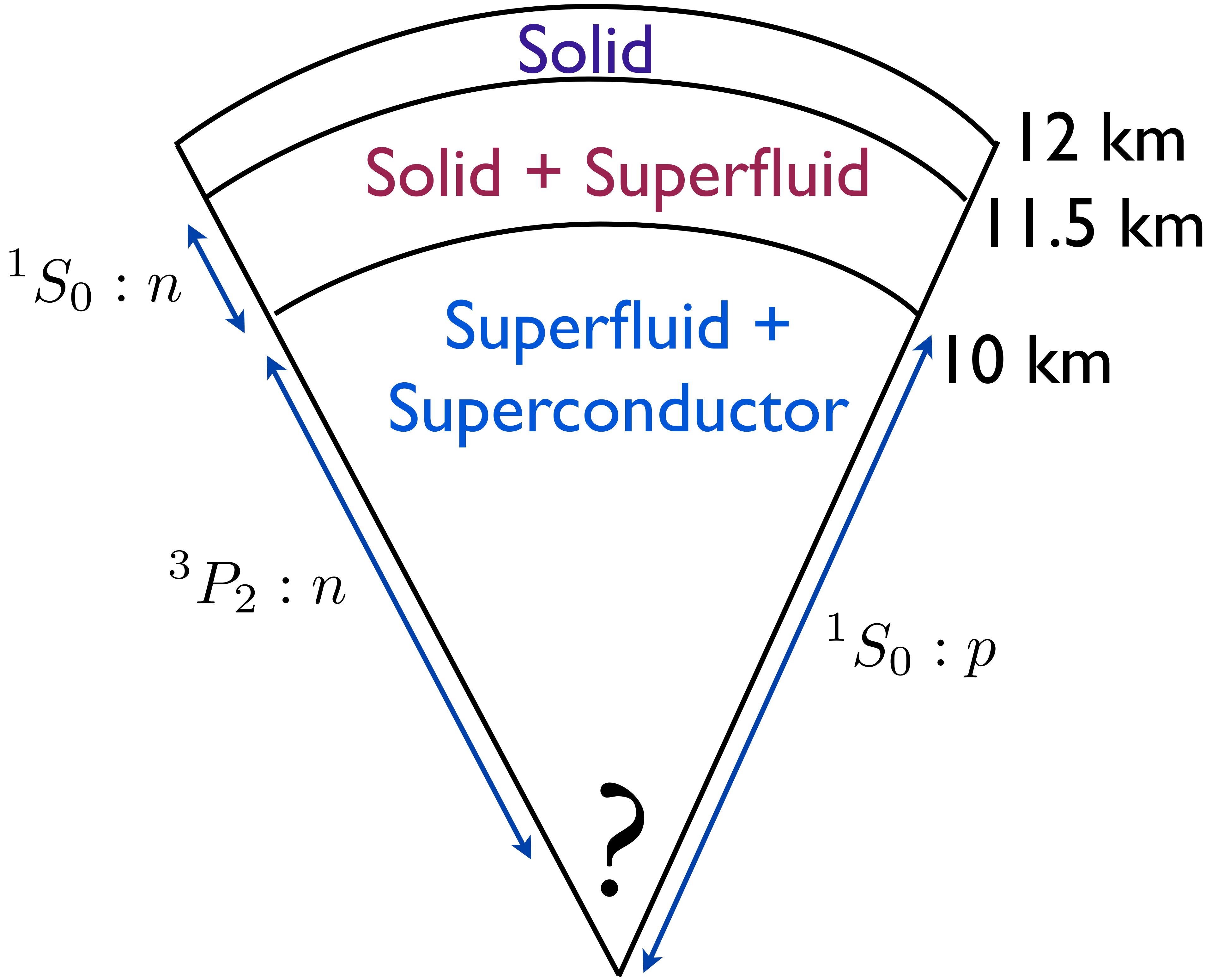
Energy gap for fermions:

$$E(p) = \sqrt{(\frac{p^2}{2M} - \mu)^2 + \Delta^2}$$

New collective mode:
Superfluid Phonon

$$\omega(k) = v_s\ k$$

A Frozen (Vanilla) Neutron Star



The nucleon degree of freedom may be frozen everywhere in a cold neutron star !

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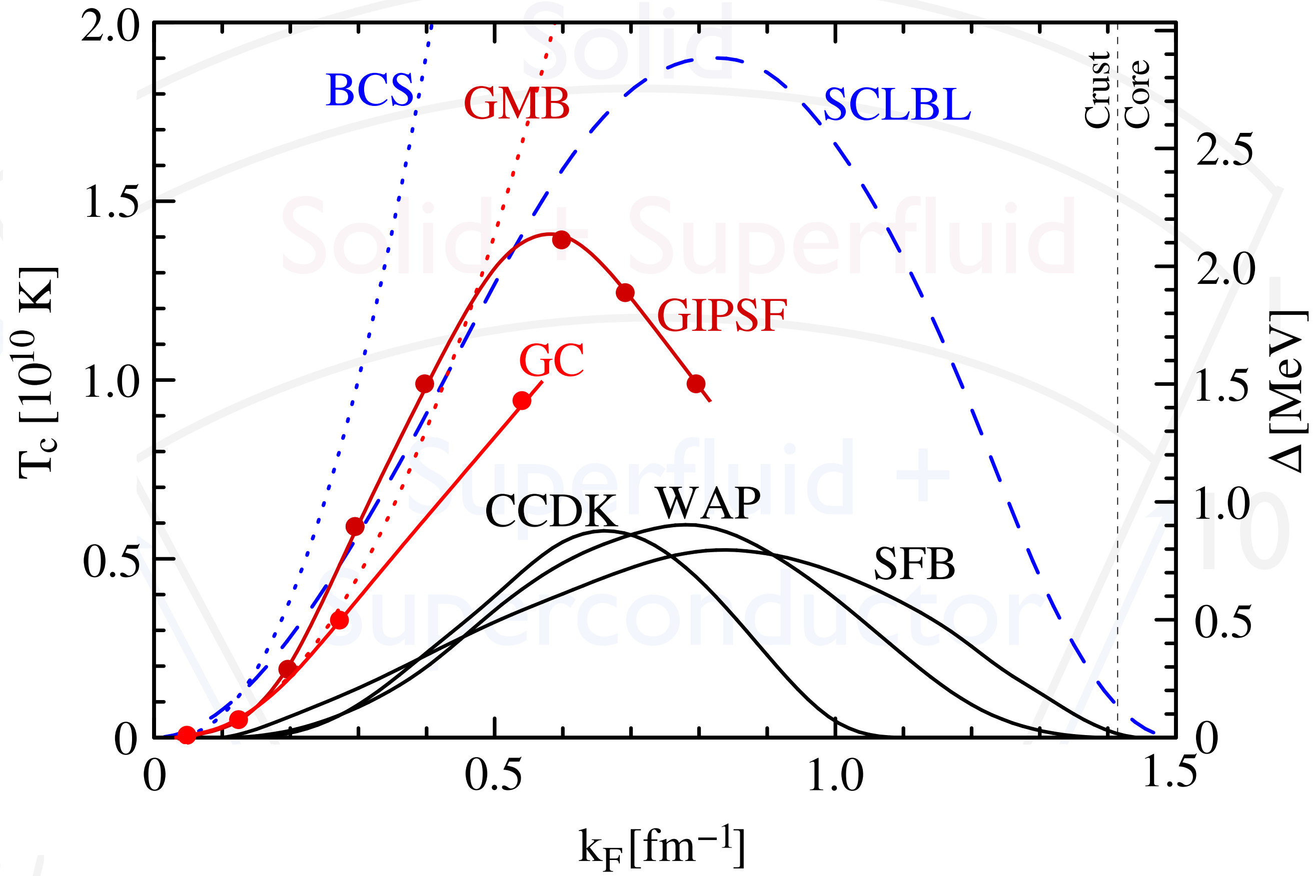


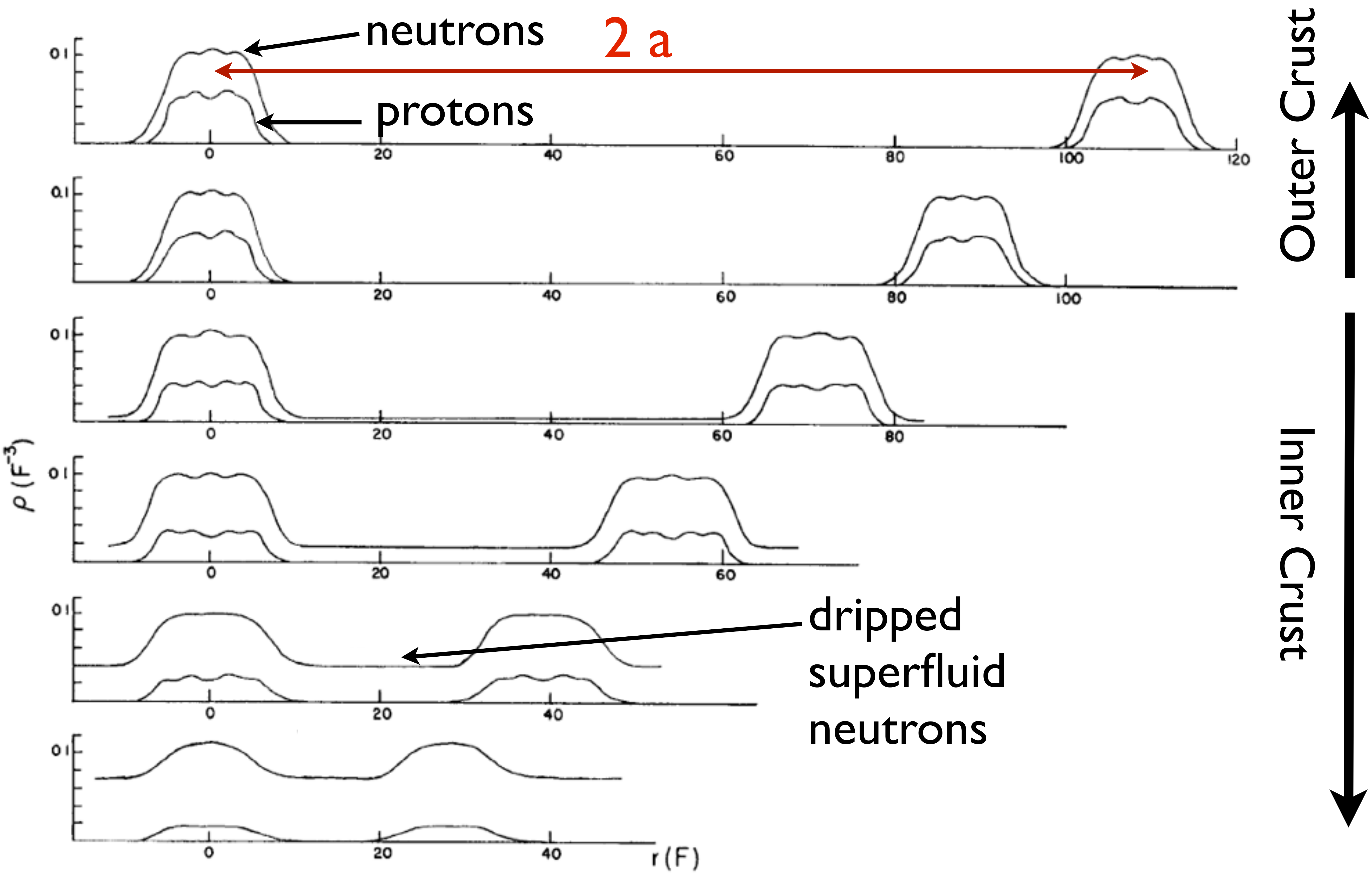
The nucleon degree of freedom may be frozen everywhere in a cold neutron star !

Transport properties dominated by

- Outer crust: Electrons and lattice phonons.
- Inner crust: Electrons, lattice phonons and superfluid phonons.
- Core: Electrons, superfluid phonons, and angulons (Goldstone bosons associated with breaking rotational symmetry).

This is good news. Describing low energy properties of dense Fermi liquids is hard ! Low energy theory of phonons is simpler.

Microscopic Structure of the Crust



Baym Pethick & Sutherland (1971) Negele & Vautherin (1973)

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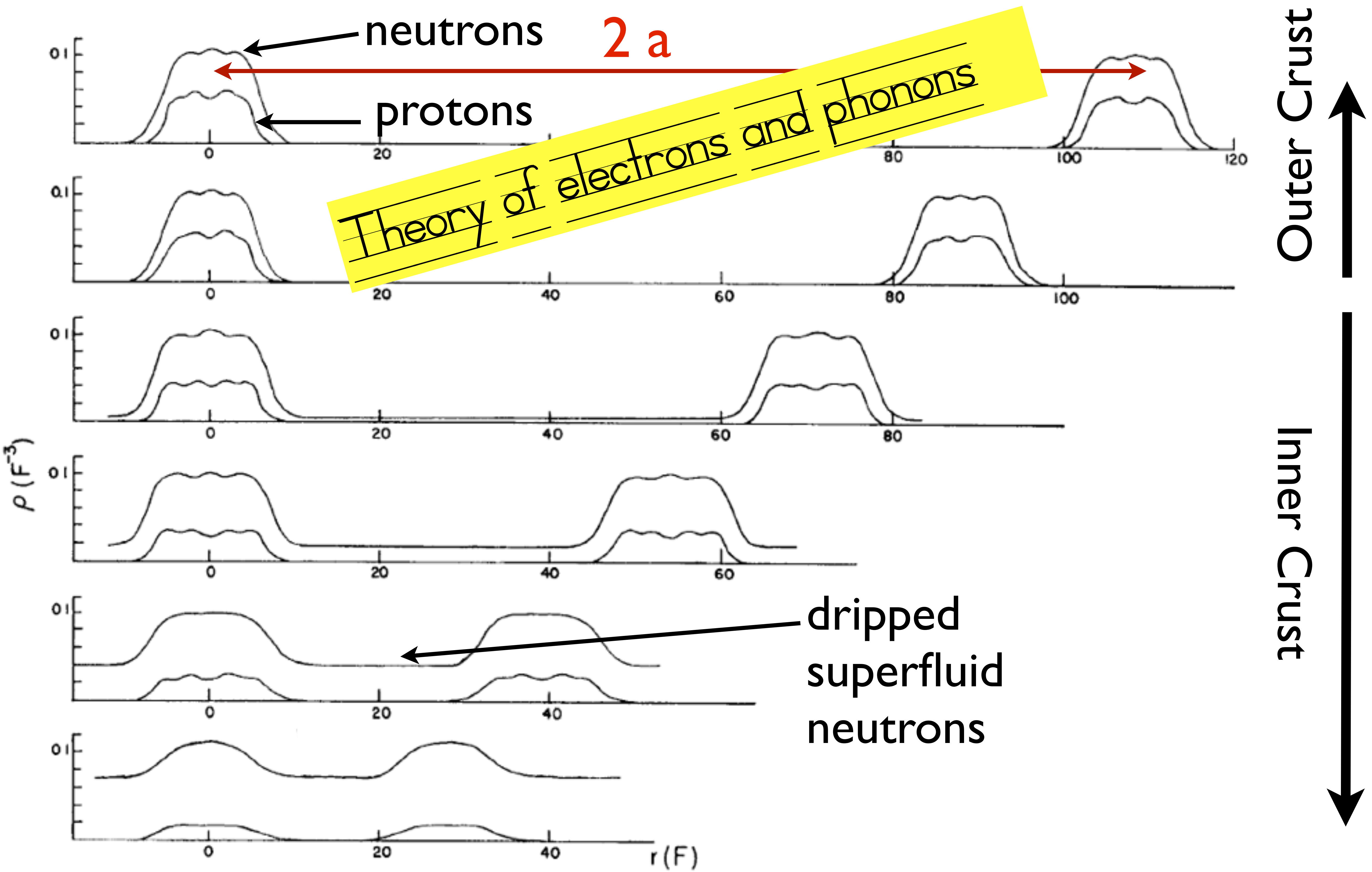

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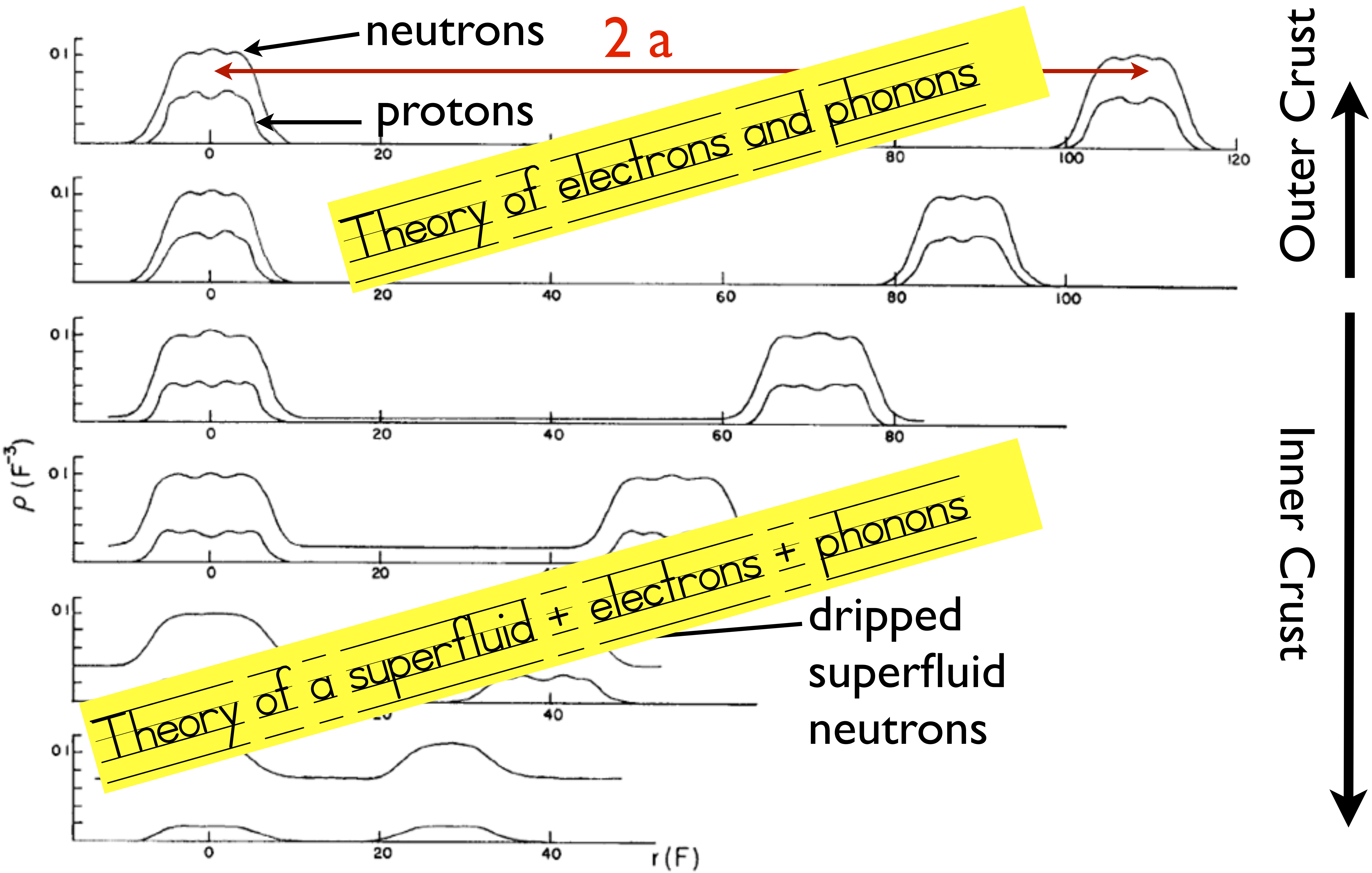

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Electrons are (nearly) free

- Electrons are dense,degenerate and relativistic.

$$n_e = Z\ n_I \qquad k_{\rm Fe} \approx E_{\rm Fe} \simeq 25 - 75\ {\rm MeV} \gg m_e$$

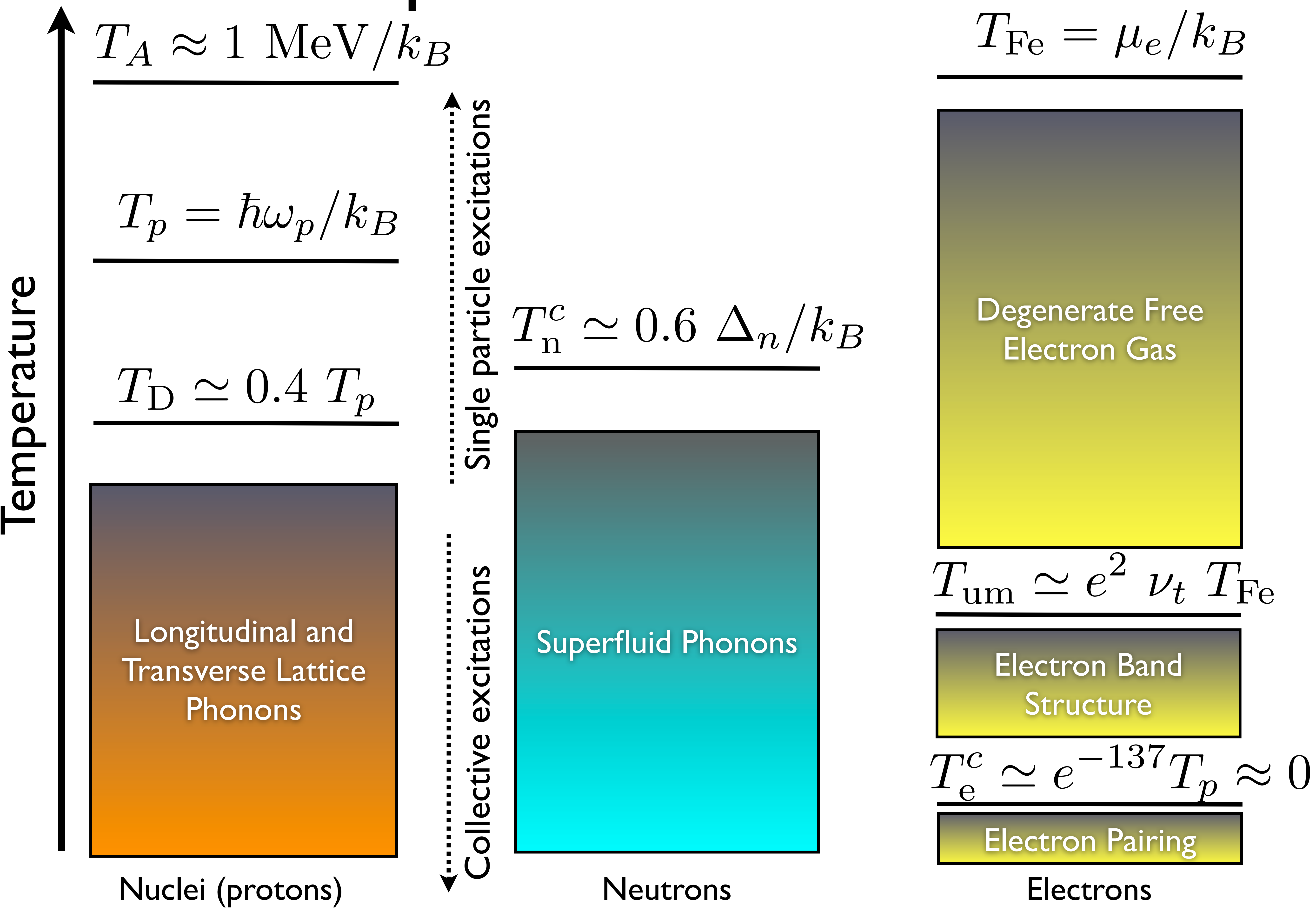
- Band gaps are small and restricted to small patches in the Fermi surface.

$$\frac{V_{\rm e-i}}{E_{\rm Fe}} \simeq \alpha_{\rm em}\ Z^{2/3} \ll 1 \qquad \frac{\delta_{\rm e}}{E_{\rm Fe}} \simeq \frac{4\alpha_{\rm em}}{3\pi} \approx 10^{-3}$$

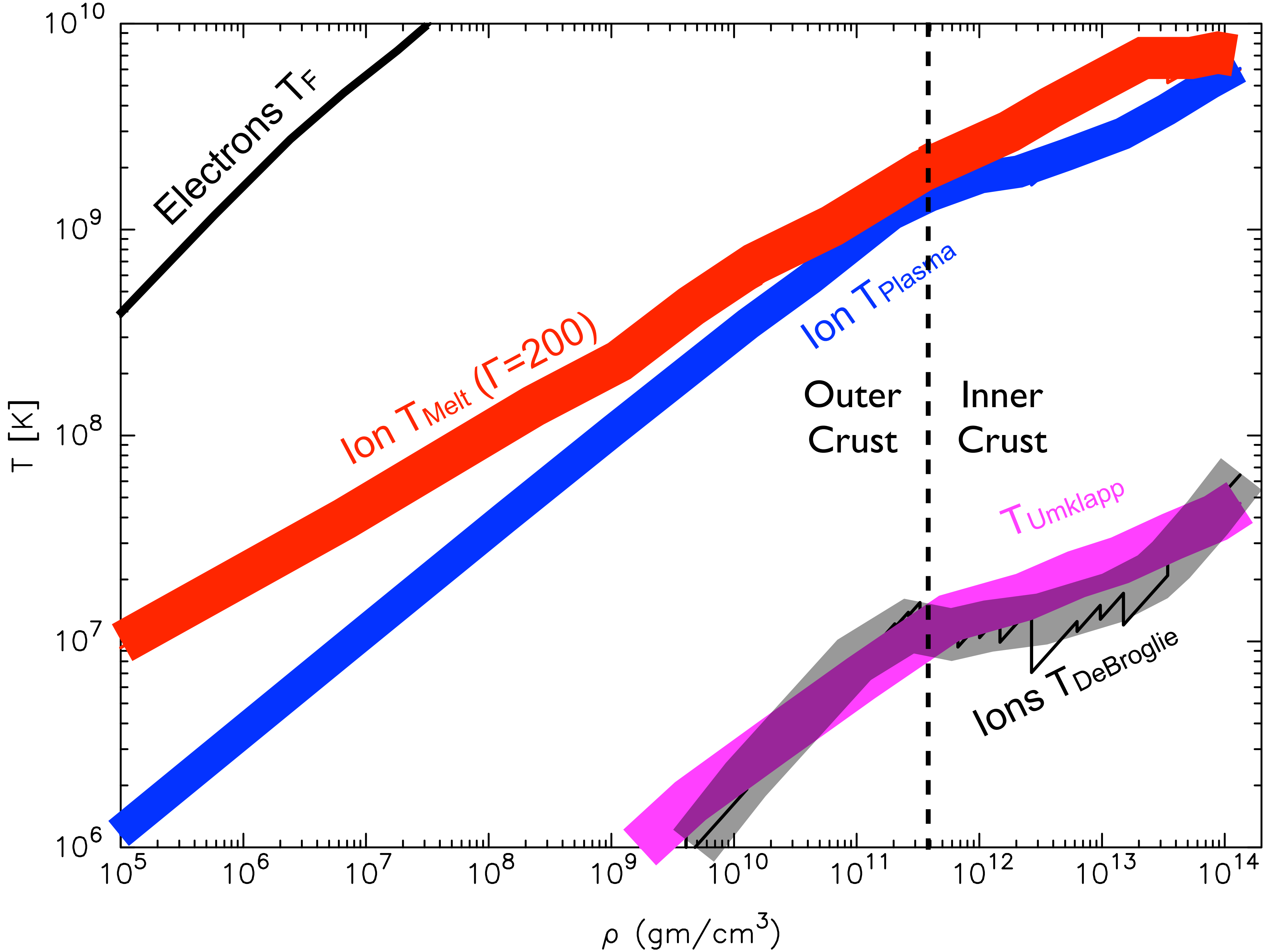
- Pairing energy is negligible.

$$T_c \simeq \omega_p^{\rm ion}\ \exp\left(-\frac{v_{Fe}}{\alpha_{\rm em}}\right) \approx 0$$

Separation of Scales



Relevant Temperature Scales in the Crust



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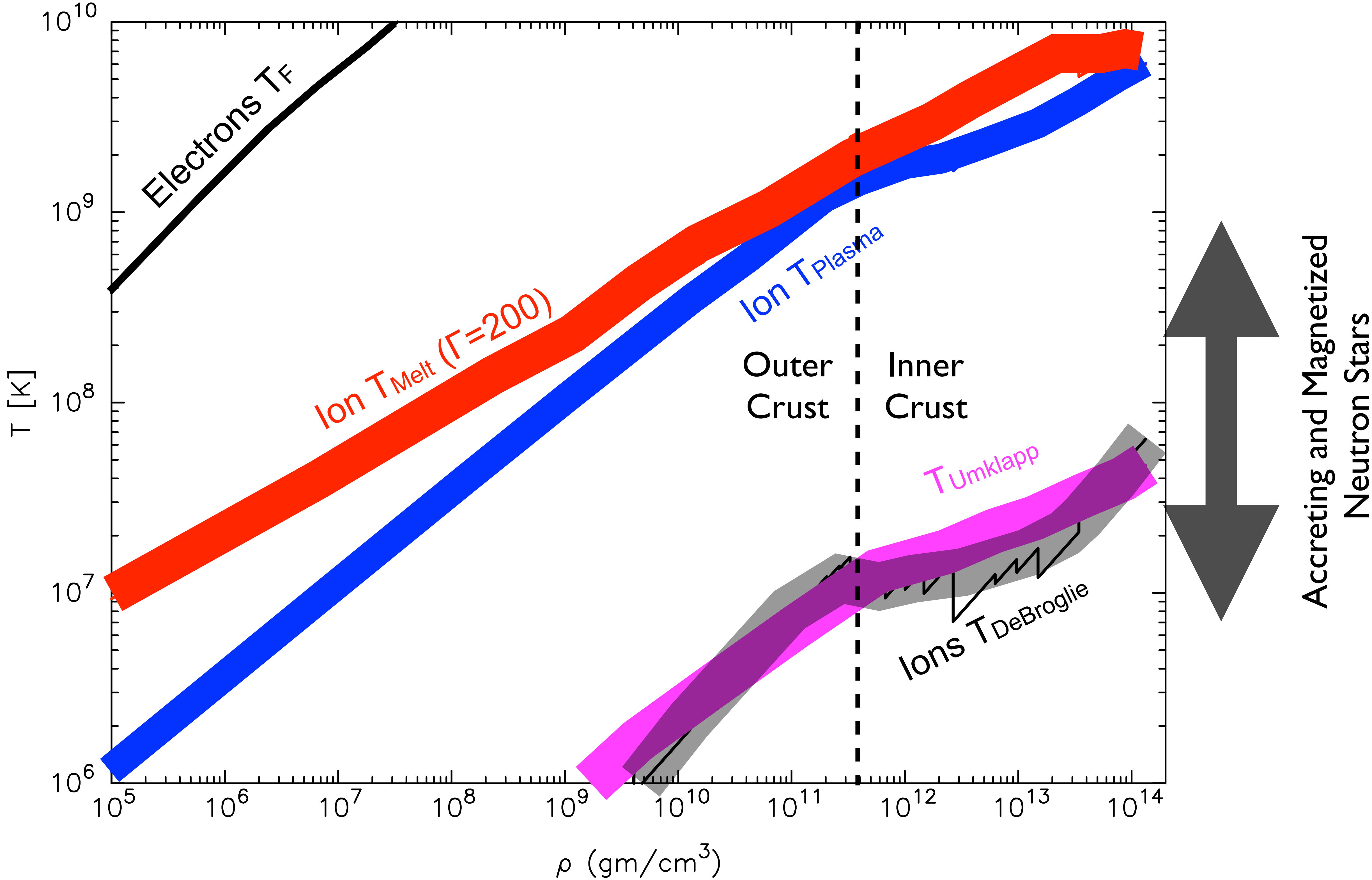

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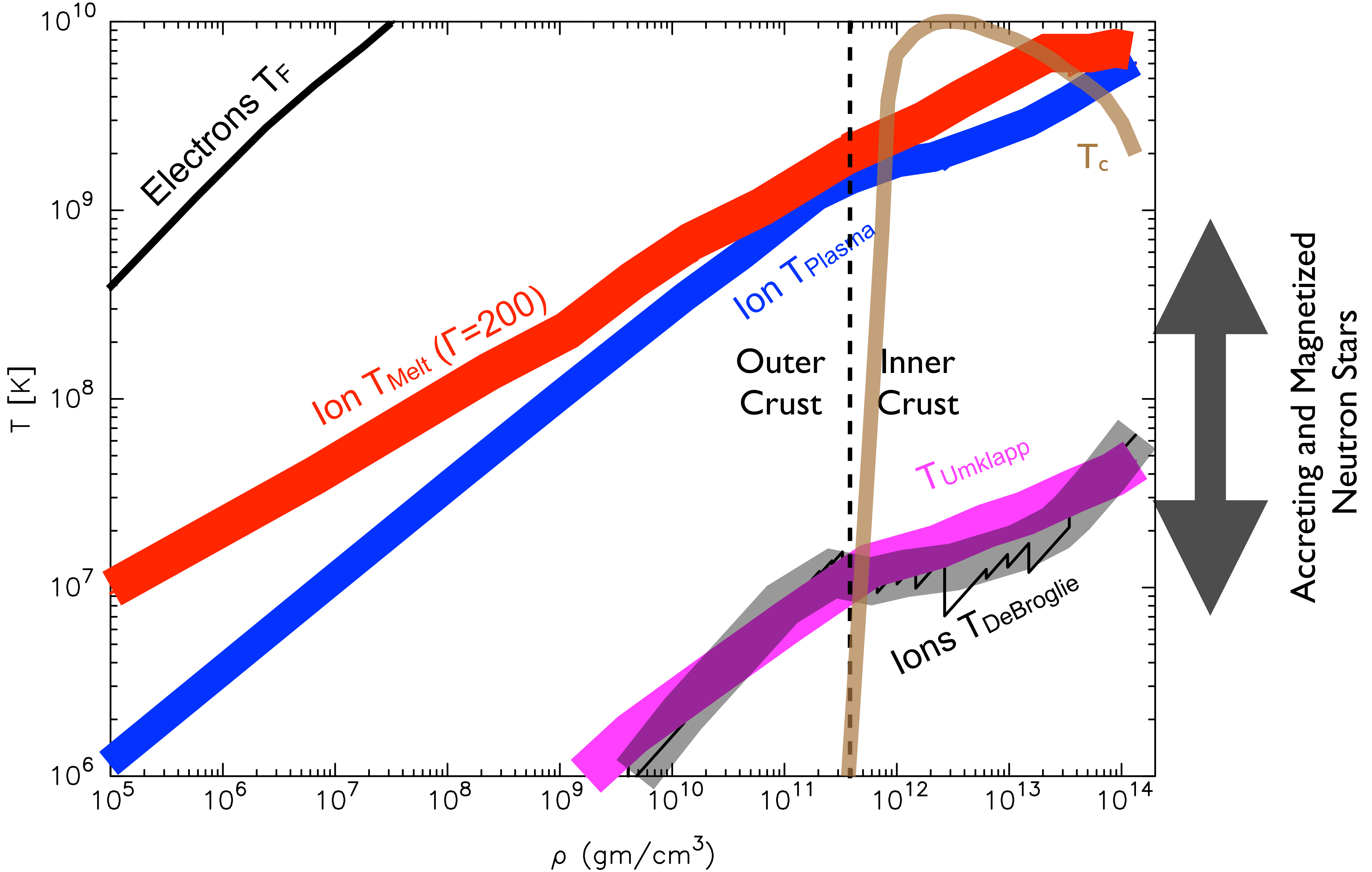

Low Energy Theory of Phonons

Proton (clusters) move collectively on lattice sites. Displacement is a good coordinate.

Neutron superfluid: Goldstone excitation is the phase of the condensate.

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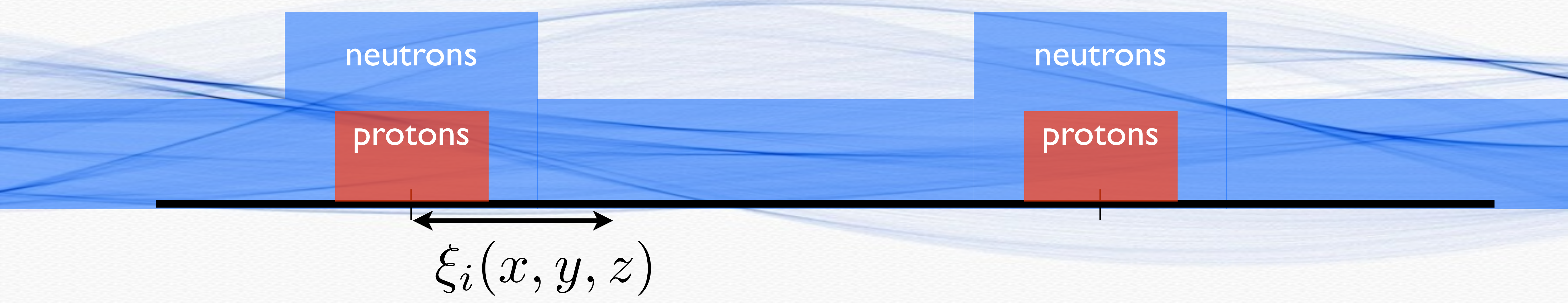

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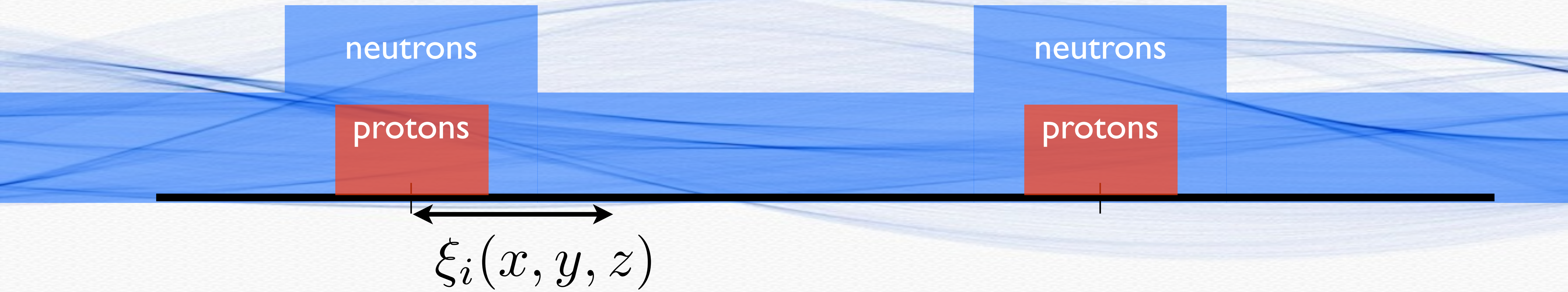

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Neutron superfluid: Goldstone excitation is the phase of the condensate.

$$\langle \psi_\uparrow(r)\psi_\downarrow(r)\rangle = |\Delta| \; \exp\left(-2i\;\theta\right)$$

"coarse-grain"

Collective coordinates:

Vector Field: $\xi_i(r,t)$
Scalar Field: $\phi(r,t)$

Symmetries & Derivative Expansion

The low energy theory must respect symmetries of the underlying Hamiltonian

$$\left\{ \begin{array}{l} \xi^{a=1..3}(\mathbf{r},t) \rightarrow \xi^{a=1..3}(\mathbf{r},t) + a^{a=1..3} \\ \phi(\mathbf{r},t) \rightarrow \phi(\mathbf{r},t) + \theta \end{array} \right.$$

Only derivative terms are allowed. Lagrangian density for the phonon system with cubic symmetry:

$$\begin{aligned} \mathcal{L} = & \frac{f_\phi^2}{2}(\partial_0\phi)^2 - \frac{v_\phi^2 f_\phi^2}{2}(\partial_i\phi)^2 + \frac{\rho}{2}\partial_0\xi^a\partial_0\xi^a - \frac{1}{4}\mu(\xi^{ab}\xi^{ab}) - \frac{K}{2}(\partial_a\xi^a)(\partial_b\xi^b) \\ & - \frac{\alpha}{2}\sum_{a=1..3}(\partial_a\xi^a\partial_a\xi^a) + g_{\text{mix}} f_\phi\sqrt{\rho}\;\partial_0\phi\partial_a\xi^a + \cdots \quad , \end{aligned}$$

where $\quad \xi^{ab} = (\partial_a\xi^b + \partial_b\xi^a) - \frac{2}{3}\partial_c\xi^c\delta^{ab}$

Cirigliano, Reddy, Sharma 2011

Identifying the Low Energy Constants

- LECs must be related to thermodynamic properties.

- Each gradient produces a unique deformation of the ground state.

- The energy cost associated with these (small) deformations provide the LECs.

For a rigorous derivation of LECs in terms of thermodynamic derivatives see arXiv:1102.5379

Cirigliano, Reddy, Sharma 2011

Inner Crust EFT

Protons:

$$\mathcal{L}_p = \frac{1}{2} n_p m \; \partial_t \xi_i \; \partial_t \xi_i - \frac{1}{2} \; K \; \partial_i \xi_i \partial_i \xi_i - \frac{1}{4} \; \mu_s \; \xi_{ij} \; \xi_{ij} + \cdots$$

$$\xi_{ij} = \partial_i \xi_j + \partial_j \xi_i - \frac{2}{3} \delta_{ij} \; \partial_k \xi_k$$

Compressibility

$$K \propto \frac{\partial^2 E}{\partial n_p \partial n_p}$$

Shear Modulus

$$\mu_s \propto \frac{Z^2 \; e^2}{a^4}$$

Neutrons:

$$\langle \psi_\uparrow(r) \psi_\downarrow(r) \rangle = |\Delta| \; \exp\left(-2i \; \theta\right)$$

$$\theta = \mu_n \; t - \; \phi$$

Ground-state

Fluctuations (Superfluid Phonons)

$$\mathcal{L}_0(X_0) = P(\mu_n) \qquad X_0 = (\partial_\mu \phi + A_\mu)(\partial^\mu \phi + A^\mu)$$

Son & Wingate (2006)

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Son & Wingate (2006)

Coupling Neutrons and Protons.
(or the superfluid and the lattice)

$$\boxed{\mathcal{L}_n = P(\mu_n) + \frac{\partial P}{\partial \mu_n}\ \delta\mu_n + \frac{1}{2}\frac{\partial^2 P}{\partial \mu_n \partial \mu_n}\ \delta\mu_n^2 + \cdots}$$

Gibbs-Duhem Relation:

$$\delta\mu_n = E_{nn}\ \delta n_n + E_{\mathrm{np}}\delta n_p$$

$$\delta\mu_n = -\partial_t\phi - E_{\mathrm{np}} n_p \partial_i \xi_i$$

density-density interaction

Velocities and current-current coupling :

$$\delta\mu_n = -\frac{(\partial_i\phi)^2}{2m} + \frac{1}{2}\gamma\ m\ (\vec{v}_n - \vec{v}_p)^2$$

current-current interaction

$$\vec{v}_n = \frac{\partial_i\phi}{m} \qquad \vec{v}_p = \partial_t\xi_i$$

Coupling Neutrons and Protons.
(or the superfluid and the lattice)

$$\boxed{\mathcal{L}_n = P(\mu_n) + n_n\ \delta\mu_n + \frac{1}{2}\chi_n \delta\ \mu_n^2 + \cdots}$$

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The Coupled System

$$\mathcal{L}_{n+p} = \frac{1}{2}(\partial_t\phi)^2 - \frac{1}{2}v_s^2\ (\partial_i\phi)^2 + \frac{1}{2}(\partial_t\xi_i)^2 - \frac{1}{2}(c_l^2 - g^2)\ (\partial_i\xi_i)^2$$
$$+\ g\ \partial_t\phi\ \partial_i\xi_i + \tilde{\gamma}\ \partial_i\phi\ \partial_t\xi_i$$

Velocities : $v_s^2 = \dfrac{n_f}{m\chi_n}$ $\quad c_l^2 = \dfrac{K + 4\mu_s/3}{m(n_p + n_b)}$

Entrainment: protons drag neutrons.

$\left\{\begin{array}{l}\text{Bound neutrons:}\quad n_b = \gamma\ n_n \\ \text{Free neutrons:}\quad n_f = n_n\ (1-\gamma)\end{array}\right.$

Longitudinal lattice phonons and superfluid phonons are coupled:

$$g = n_p\ E_{np}\ \sqrt{\frac{\chi_n}{m(n_p + n_b)}} \qquad \tilde{\gamma} = \frac{-n_b\ v_s}{\sqrt{(n_p + n_b)n_f}}$$

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Transverse lattice phonons:

$$\mathcal{L}_t = \frac{1}{2}(\partial_t\xi_i)^2 - \frac{1}{2}c_t^2(\partial_i\xi_j + \partial_j\xi_i)^2 \quad \Rightarrow \quad c_t^2 = \frac{\mu_s}{m(n_p+n_b)}$$

Low energy constants

c_l	c_t	v_s	g	$\tilde{\gamma}$
K	μ_s	E_{nn}	E_{np}	n_b

Thermodynamic Derivatives:

$$E_{\mathrm{nn}} = \frac{\partial^2 E}{\partial n_n \partial n_n} \qquad E_{\mathrm{pp}} = \frac{\partial^2 E}{\partial n_p \partial n_p} \qquad E_{\mathrm{np}} = \frac{\partial^2 E}{\partial n_n \partial n_p}$$

Phonon mixing and drag

$$\mathcal{L}_{\rm sPh-lPh} = g\ \partial_0\phi\ \partial_i\xi_i + \gamma\ \partial_i\phi\ \partial_0\xi_i$$

density-density interaction:

$$g = -\ \frac{n_p\ v_\phi}{\sqrt{n_n^c(n_p + n_n^b)}}\frac{\partial n_n}{\partial n_p}$$

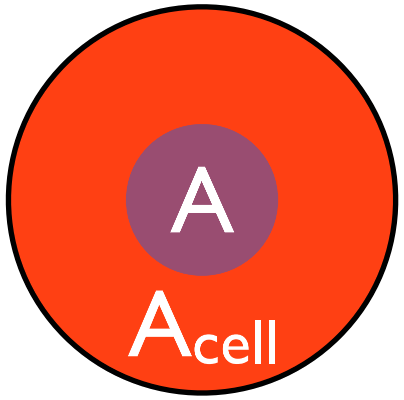
velocity-velocity interaction:

$$\gamma = \frac{n_n^b\ v_\phi}{\sqrt{n_n^c(n_p + n_n^b)}}$$

Entrainment

Chamel (2005)
Carter, Chamel & Haensel (2006)

$n_n^b \neq$ number of "bound" neutrons.



A=N+Z

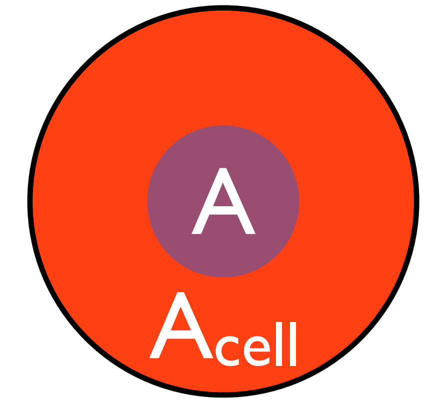
Bragg scattering off the lattice is important.

$$n_n^{\rm c} = \frac{m}{24\pi^3\hbar^2} \sum_\alpha \int_{\rm F} |\boldsymbol{\nabla}_{\boldsymbol{k}} \varepsilon_{\alpha\boldsymbol{k}}| {\rm d}\mathcal{S}^{(\alpha)}$$

$$n_n^b = n_n - n_n^c$$

Entrainment

Chamel (2005)
Carter, Chamel & Haensel (2006)



A=N+Z

$n_n^b \neq$ number of "bound" neutrons.

Bragg scattering off the lattice is important.

neutron single-particle energy

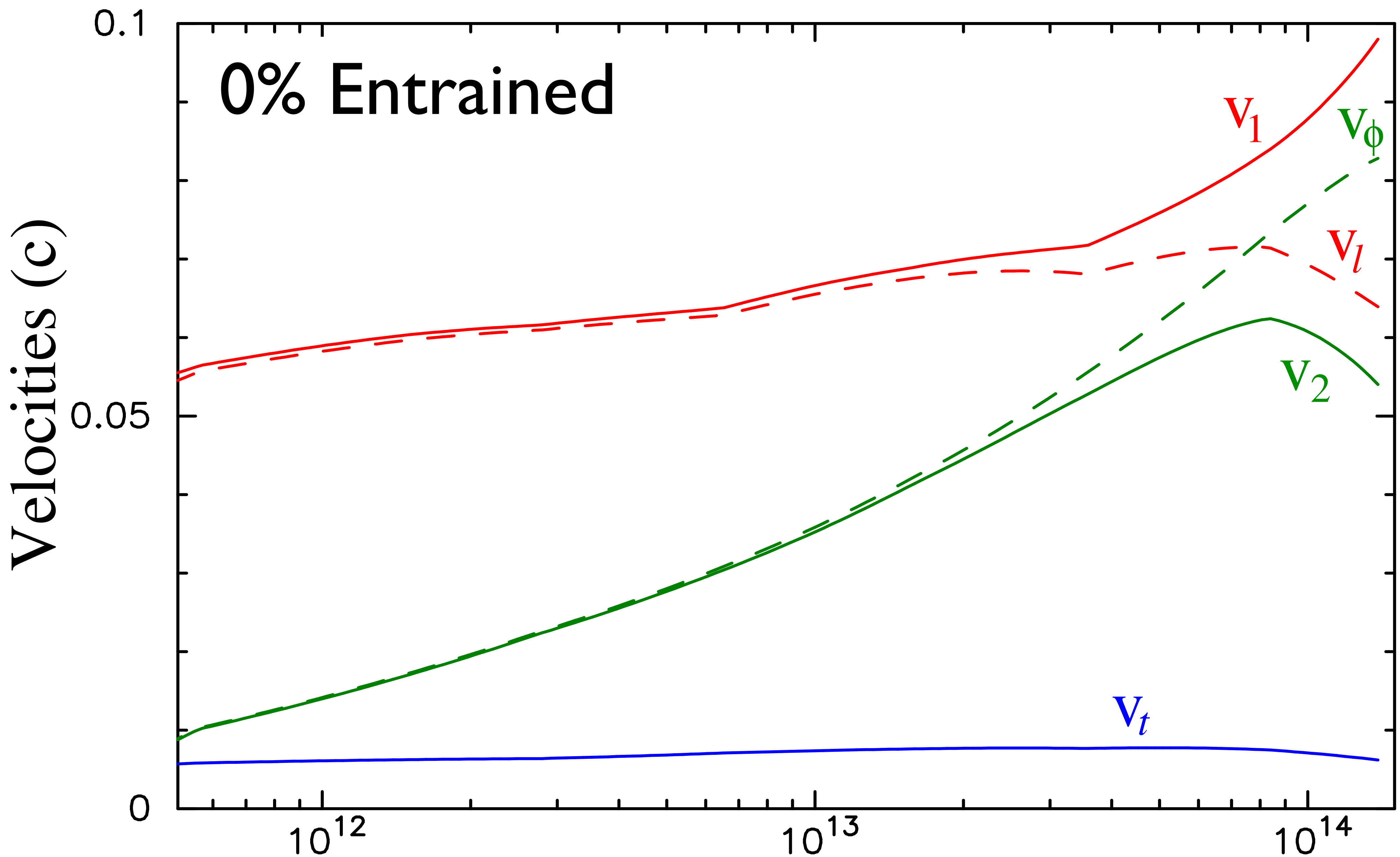
$$n_n^{\mathrm{c}} = \frac{m}{24\pi^3\hbar^2} \sum_{\alpha} \int_{\mathrm{F}} |\nabla_{\boldsymbol{k}} \varepsilon_{\alpha \boldsymbol{k}}| \mathrm{d}\mathcal{S}^{(\alpha)}$$

$$n_n^b = n_n - n_n^c$$

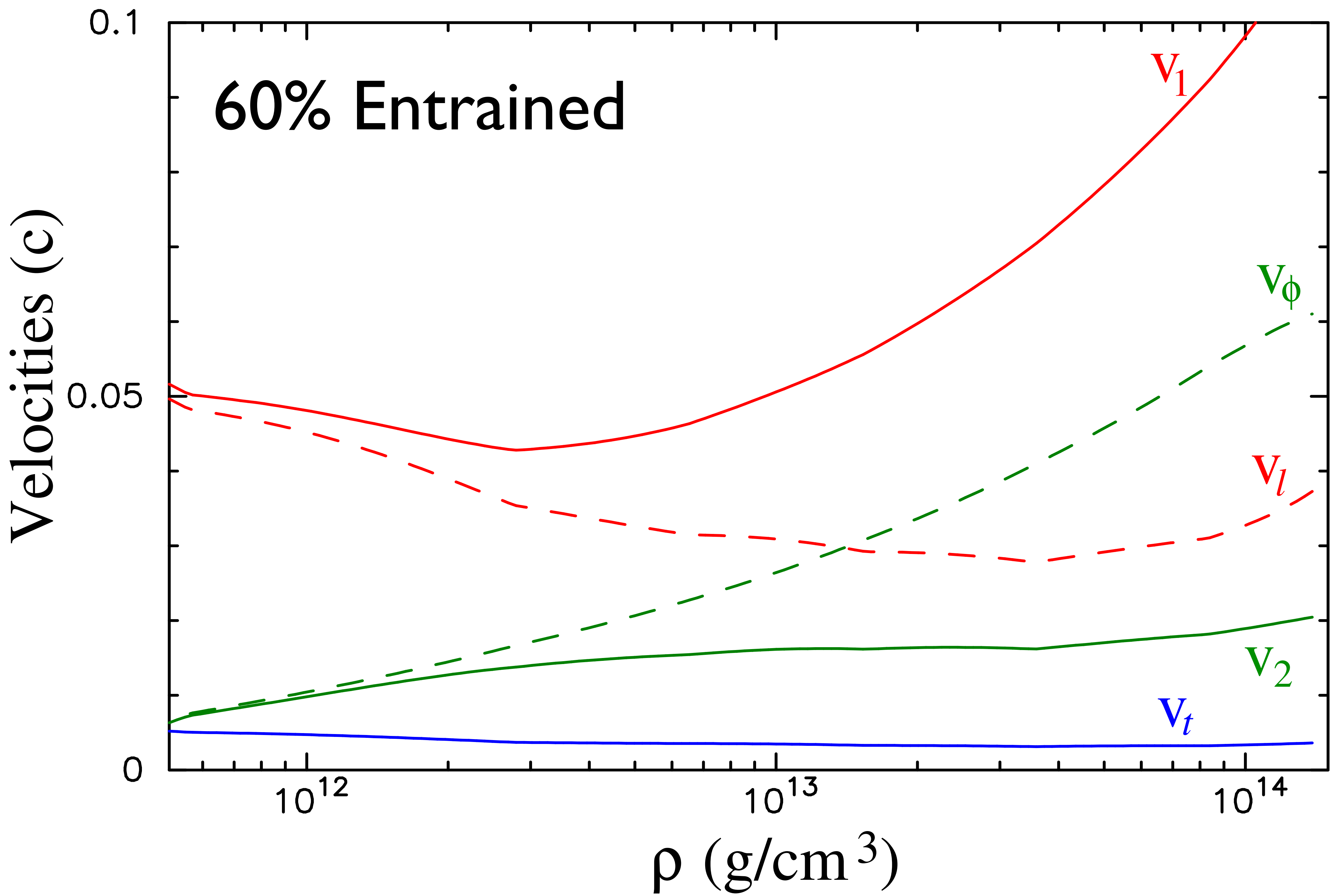
Complex interplay of nuclear and band structure effects.
The nuclear surface and disorder are likely to play a role.

Longitudinal lattice phonons and superfluid phonons are strongly coupled by entrainment.

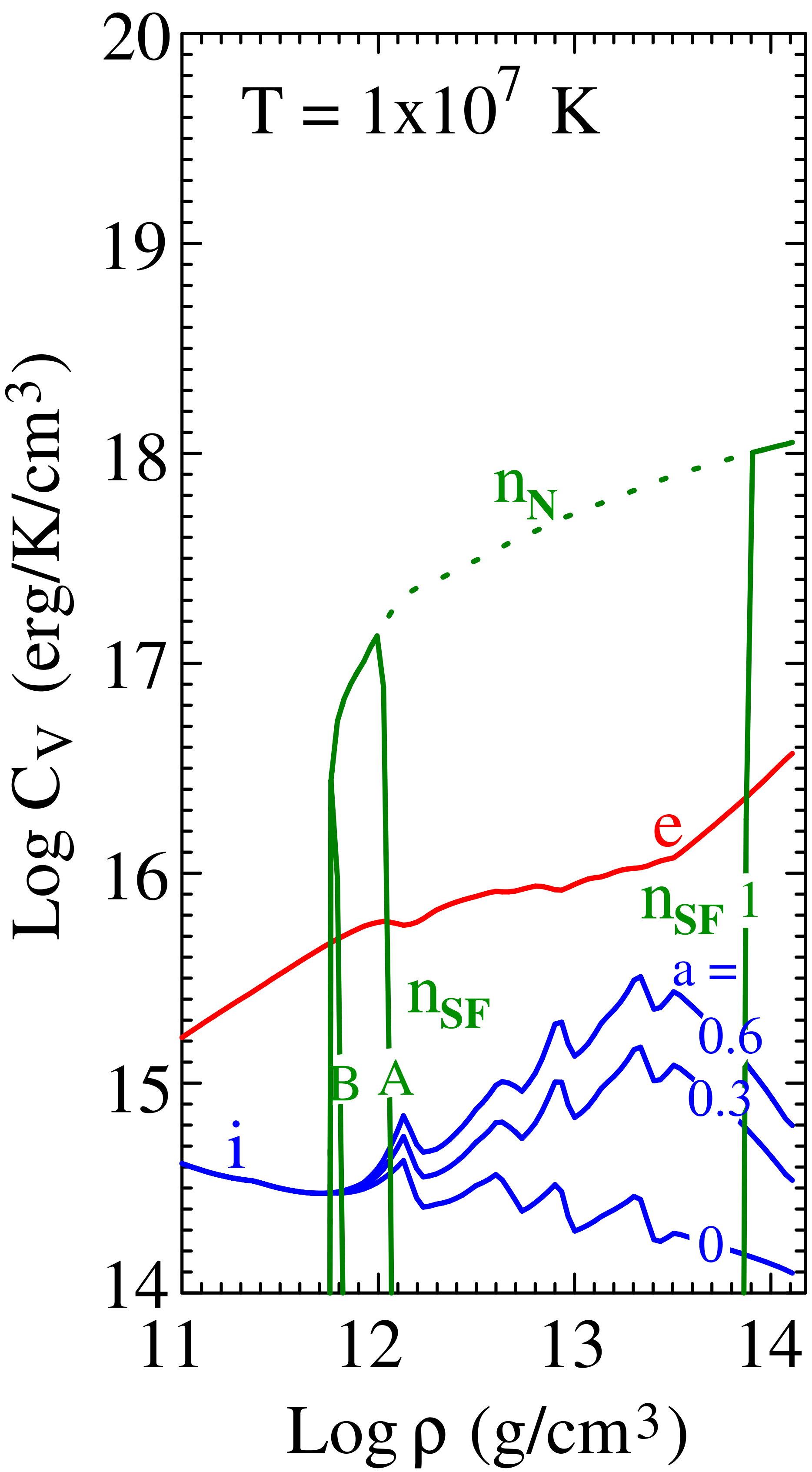
Mixed & Entrained Modes



Mixed & Entrained Modes



Crustal Specific Heat



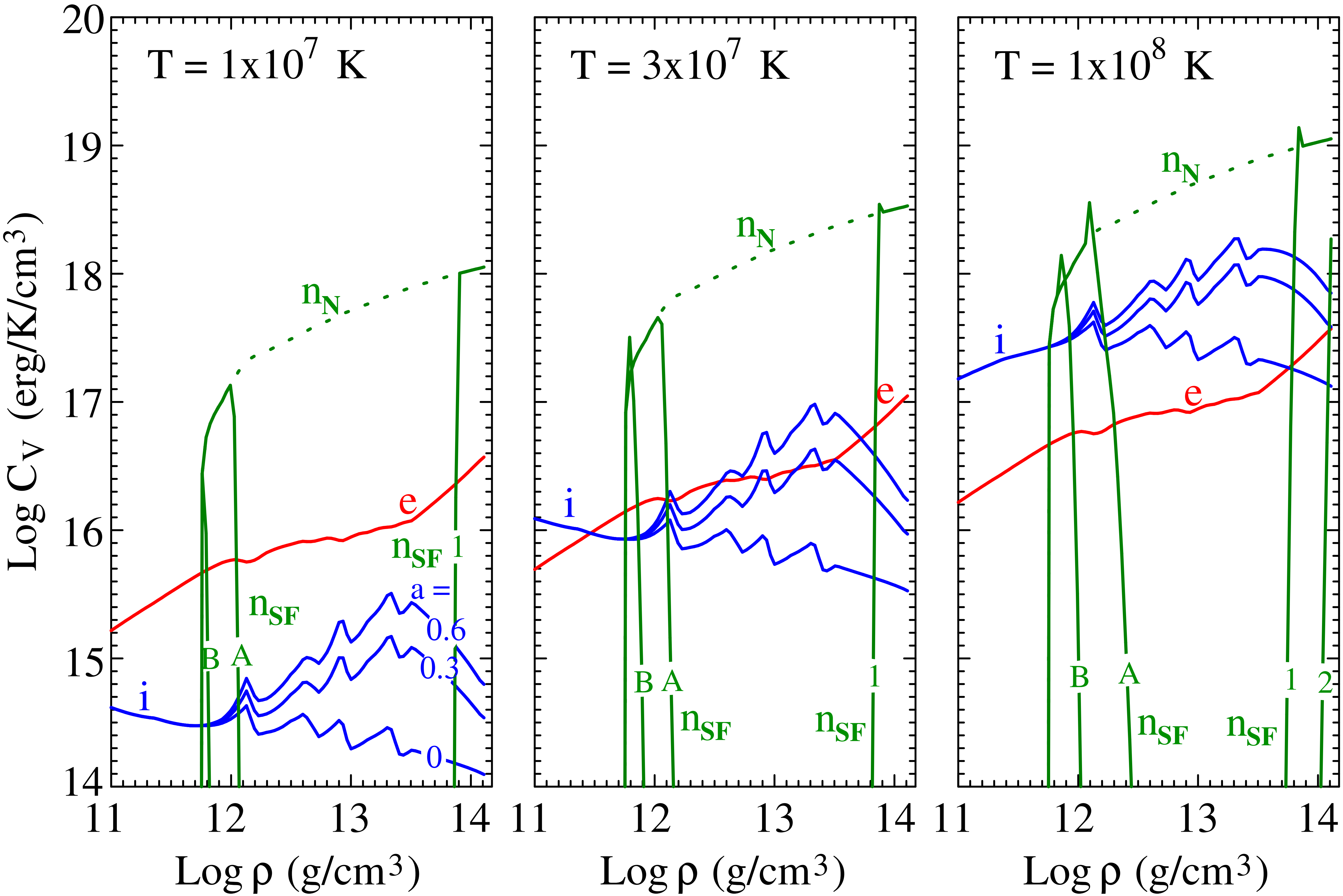
Electrons: $C_v^e = \frac{1}{3}\mu_e^2\, T$

Ions: $C_v^{\rm lph} = \frac{2\pi^2}{15}\left(\frac{T^3}{v_l^3} + \frac{2\,T^3}{v_t^3}\right)$

Neutrons:
$$\begin{cases} C_v^{\rm sph} = \frac{2\pi^2}{15}\frac{T^3}{v_\phi^3} & (T \ll T_c) \\ C_v^{\rm neutron} = \frac{1}{3}\, m_n\, k_{\rm Fn}\, T & (T > T_c) \end{cases}$$

Page & Reddy (2012)

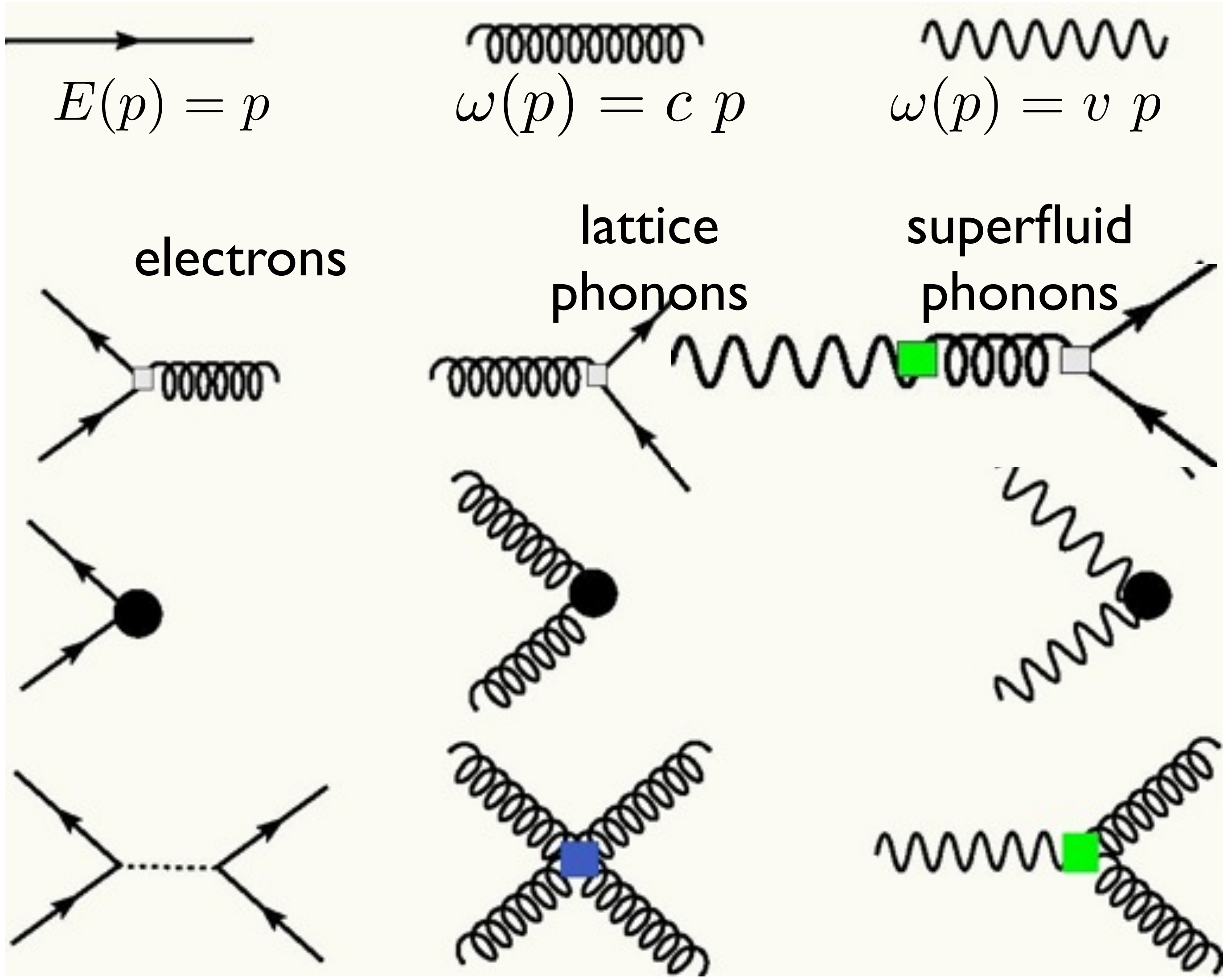
Crustal Specific Heat



Page & Reddy (2012)

Transport: Thermal Conduction

$$\kappa = \frac{1}{3}\, C_v \times v \times \lambda$$

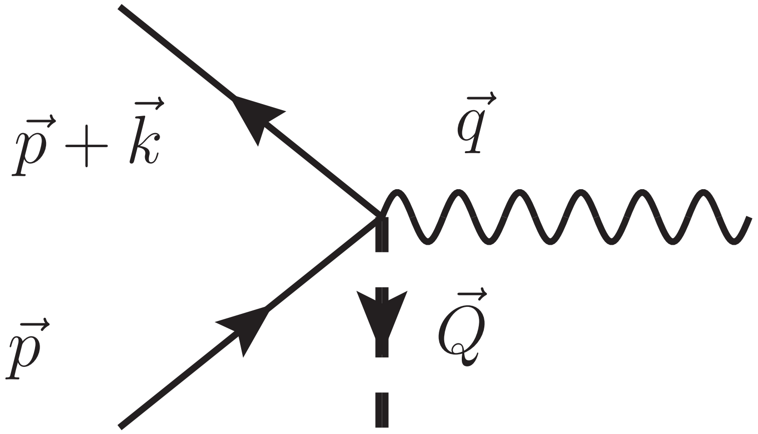


Cirigliano, Reddy & Sharma (2011)

- Dissipative processes:

- Umklapp is important:

$$\frac{k_{\rm Fe}}{q_{\rm D}} = \left(\frac{Z}{2}\right)^{1/3} > 1$$



Flowers & Itoh (1976)

Electron Bragg scatters and emits a transverse phonon.

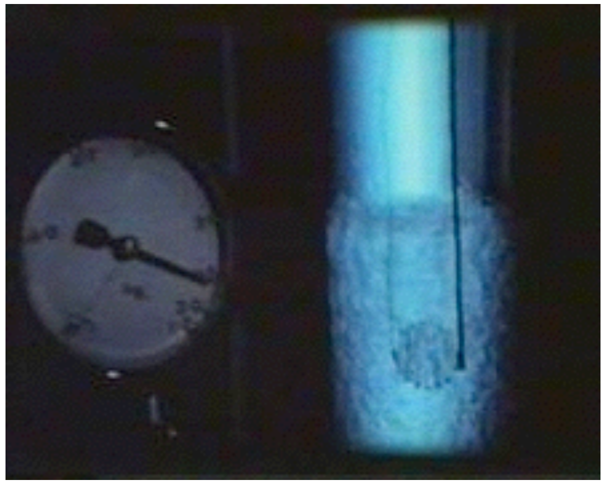
Superfluid Conduction

Photographs: JF Allen and JMG Armitage (St Andrews University 1982).

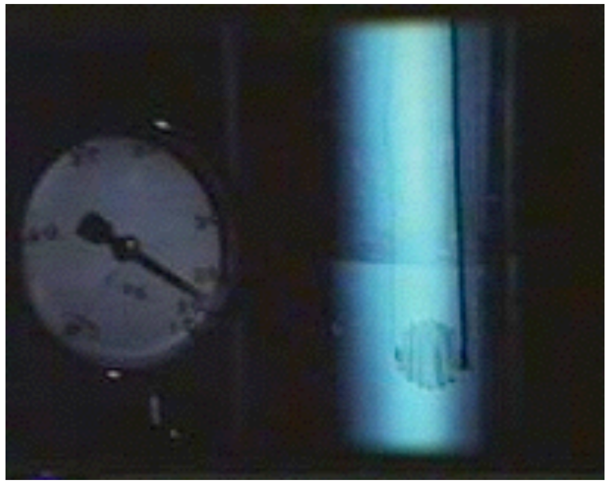
Its impossible to sustain a temperature gradient in bulk superfluid helium !

$$\vec{Q} = S^{(\mathrm{sPh})} T \vec{v}_n$$

$$S^{(\mathrm{sPh})} = \frac{1}{3} C_v^{(\mathrm{sPh})} = \frac{2\pi^2}{15\ c_s^3} T^3$$



$T>T_c$



$T<T_c$

Two fluid model: Counter-flow transports heat.
(Its the superfluid phonon fluid)

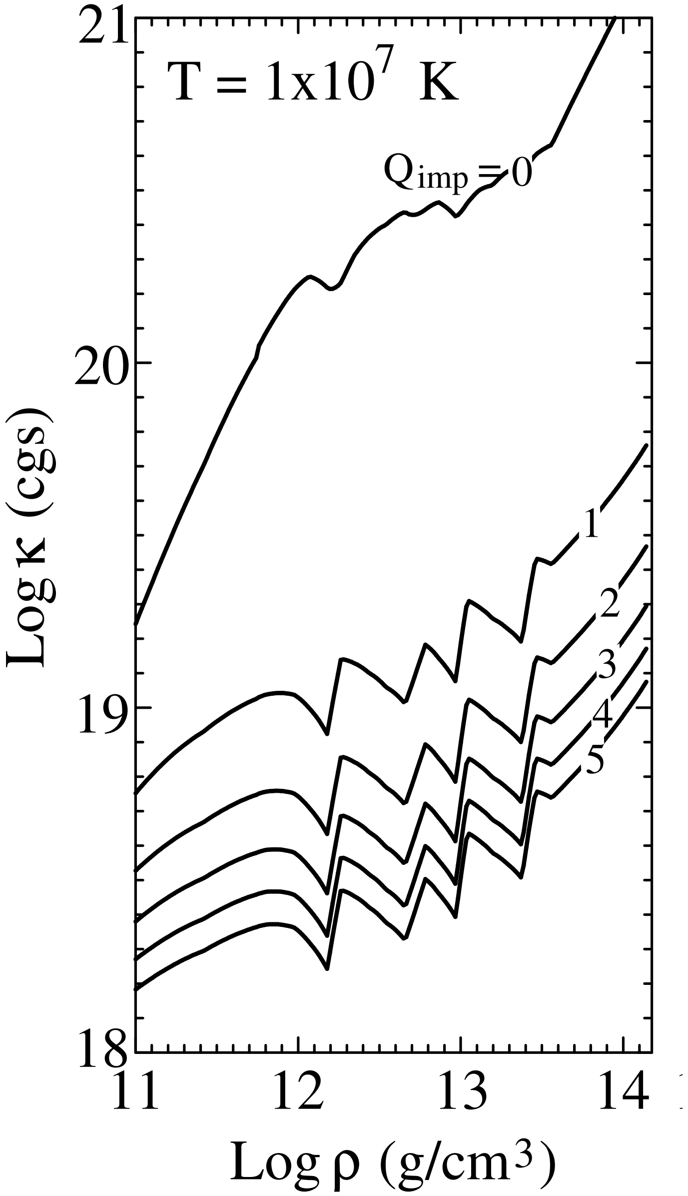
The velocity is limited only by fluid dynamics: (i) boundary shear viscosity or (ii) superfluid turbulence.

Why does this not occur in neutron stars ?
Answer: Fluid motion is damped by electrons.

Aguilera, Cirigliano, Reddy & Sharma (2009)

Electron Conduction

$$\kappa_e = \frac{1}{9}\mu_e^2\, T\, \lambda_e$$



Electron-phonon:

$$\begin{cases} \lambda_e^{\mathrm{ph}} \propto v_t^3/T^2 & T \geq T_{\mathrm{um}} \\ \lambda_e^{\mathrm{ph}} \propto v_l^4/T^3 & T \ll T_{\mathrm{um}} \end{cases}$$

$$T_{\mathrm{um}} = (4e^3/9\pi)\, v_t\, k_{\mathrm{Fe}}$$

Electron-impurity:

$$\lambda_e^{\mathrm{imp}} = \frac{3\pi\langle Z\rangle}{4e^4 Q_{\mathrm{imp}}\, k_{\mathrm{Fe}}}\, \Lambda^{-1}$$

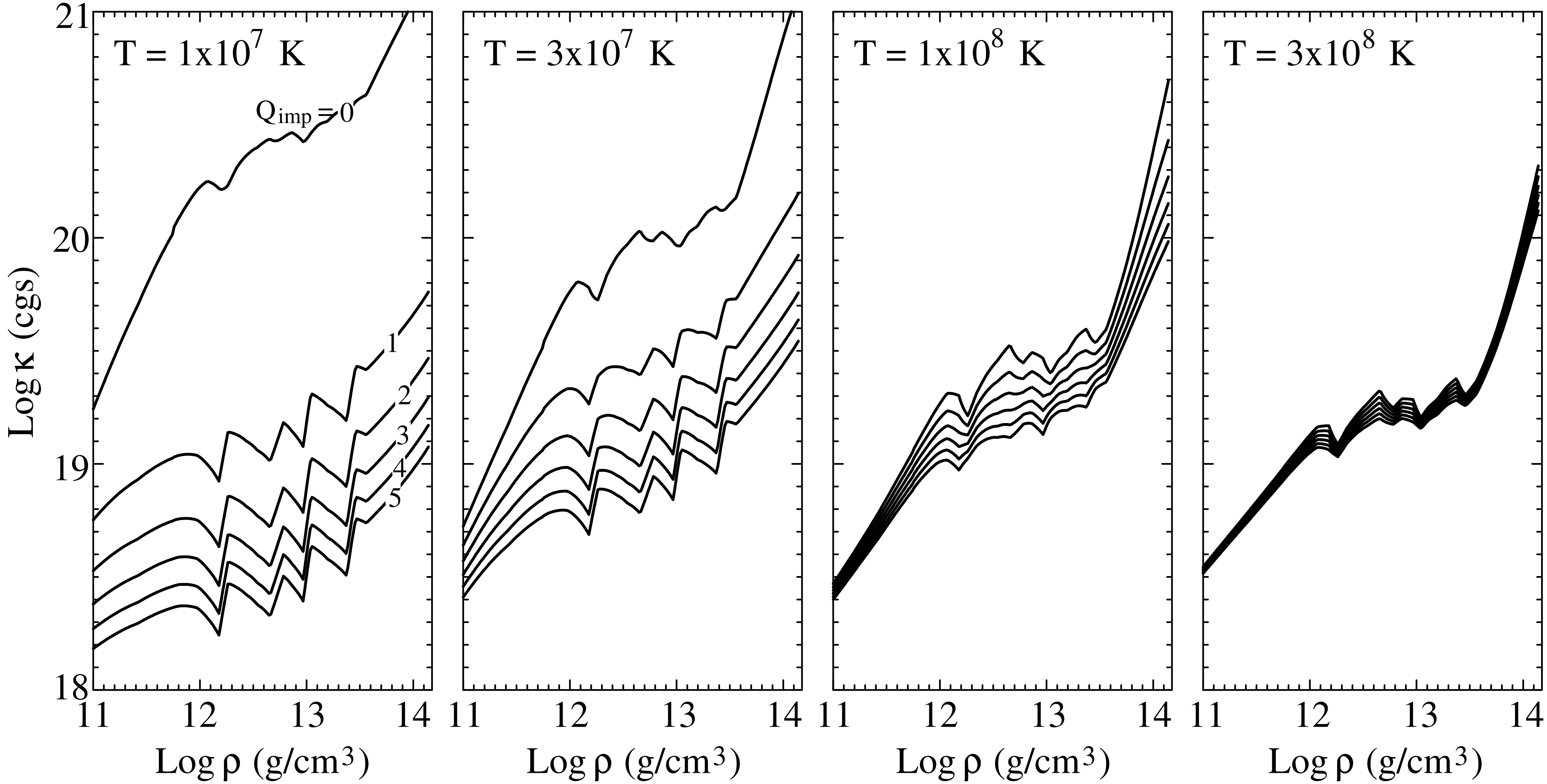
$$Q_{\mathrm{imp}} = \frac{1}{n_{\mathrm{ion}}} \sum_i n_i\, (Z_i - \langle Z\rangle)^2$$

Impurity scattering is important at low temperature.

Flowers & Itoh (1976)

Electron Conduction

$$\kappa_e = \frac{1}{9}\mu_e^2\, T\, \lambda_e$$



Impurity scattering is important at low temperature.

Flowers & Itoh (1976)

Low energy excitations in the core

Neutrons are superfluid ($T<T^n_c$): Electrons + 4 Goldstone modes (3 neutron modes and 1 electron-proton mode). Neutron condensate breaks baryon number and rotational symmetry. 2 angulons + 1 superfluid phonon. (Bedaque, Rupak, Savage, (2003), Bedaque and Reddy (2013), Bedaque, Nicholson (2013))

Neutrons are normal ($T>T^n_c$): Electrons, neutrons + 1 Goldstone boson (electron-proton mode).

Superfluid Phonons:

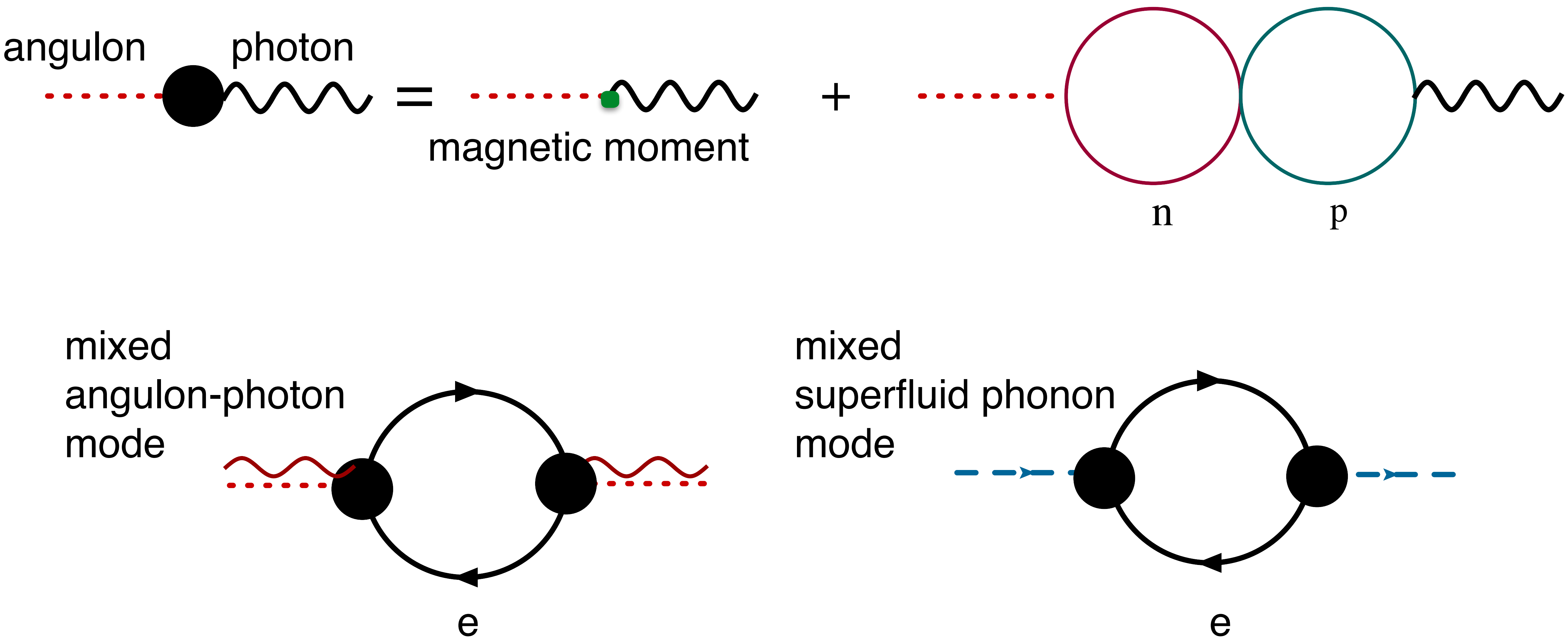
$$\mathcal{L}_{\rm phn} = \frac{1}{2}(\partial_0\phi)^2 - \frac{v_n^2}{2}(\partial_i\phi)^2 + \frac{1}{2}(\partial_0\xi)^2 - \frac{v_p^2}{2}(\partial_i\xi)^2 + v_{\rm np}^2\,\partial_0\phi\,\partial_0\xi + \frac{1}{f_{\rm ep}}\,\partial_0\xi\,\psi_{\rm e}^\dagger\,\psi_{\rm e} + \cdots\quad ,$$

Angulons:

$$\mathcal{L}_{\rm ang} = \sum_{i=1,2}\left[\frac{1}{2}(\partial_0\beta_i)^2 - \frac{1}{2}{v_\perp^i}^2((\partial_x\beta_i)^2 + (\partial_y\beta_i)^2) + v_\parallel^2(\partial_z\beta_i)^2\right. \\ + \frac{e g_n f_\beta}{2M\sqrt{-\nabla_\perp^2}}\left[\mathbf{B}_1\partial_0(\partial_y\beta_1 + \partial_x\beta_2) + \mathbf{B}_2\partial_0(\partial_x\beta_1 - \partial_y\beta_2)\right]$$

Bedaque and Reddy (2013),

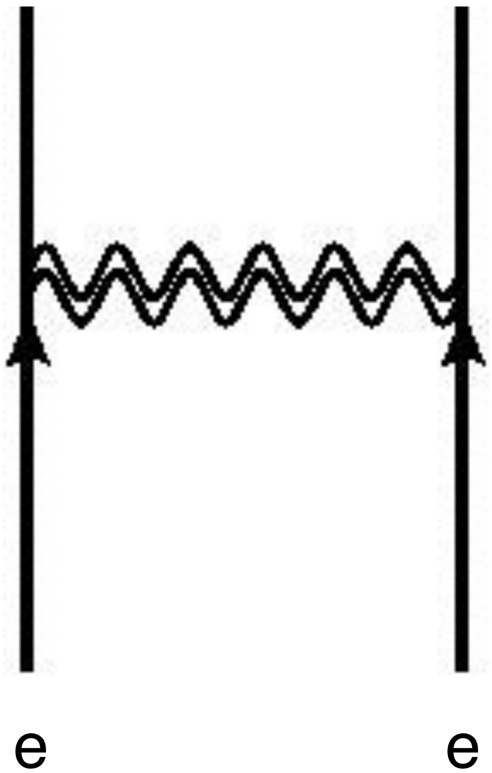
Mixing and Damping of Goldstone Bosons



Modes decay rapidly due to the coupling to the large density of electron-hole states. Do not contribute to transport.

Bedaque and Reddy (2013),

Electron Scattering in the Core



Superconducting protons:
Both electric and magnetic photon exchange is screened. Debye and Meissner screening are strong. Large suppression in scattering rates.
Normal protons:
Magnetic interaction (current-current) is dynamically screened due to Landau damping. This screening is weak. Scattering weakly suppressed.

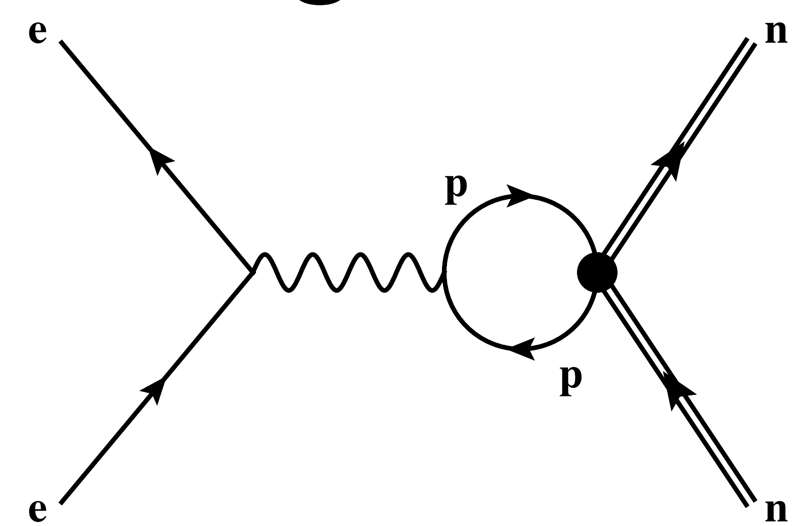
Pethick and Heiselberg (1993), Shternin and Yakovlev (2006,2007)

$$|M_{12}|^2 \propto \left| \frac{J^{(0)}_{1'1} J^{(0)}_{2'2}}{q^2 + \Pi_l} - \frac{\boldsymbol{J}_{t1'1} \cdot \boldsymbol{J}_{t2'2}}{q^2 - \omega^2 + \Pi_t} \right|^2$$

$$\Pi_t(\omega, q) \simeq \alpha_{em}\, k_{Fp}^2 \left(4\pi \frac{\Delta_p}{q} + 2i\, \frac{\omega}{q} \right)$$

Electron-Neutron Scattering

Induced interaction is strong due to a strong neutron-proton interaction. Much larger than the magnetic moment interaction.



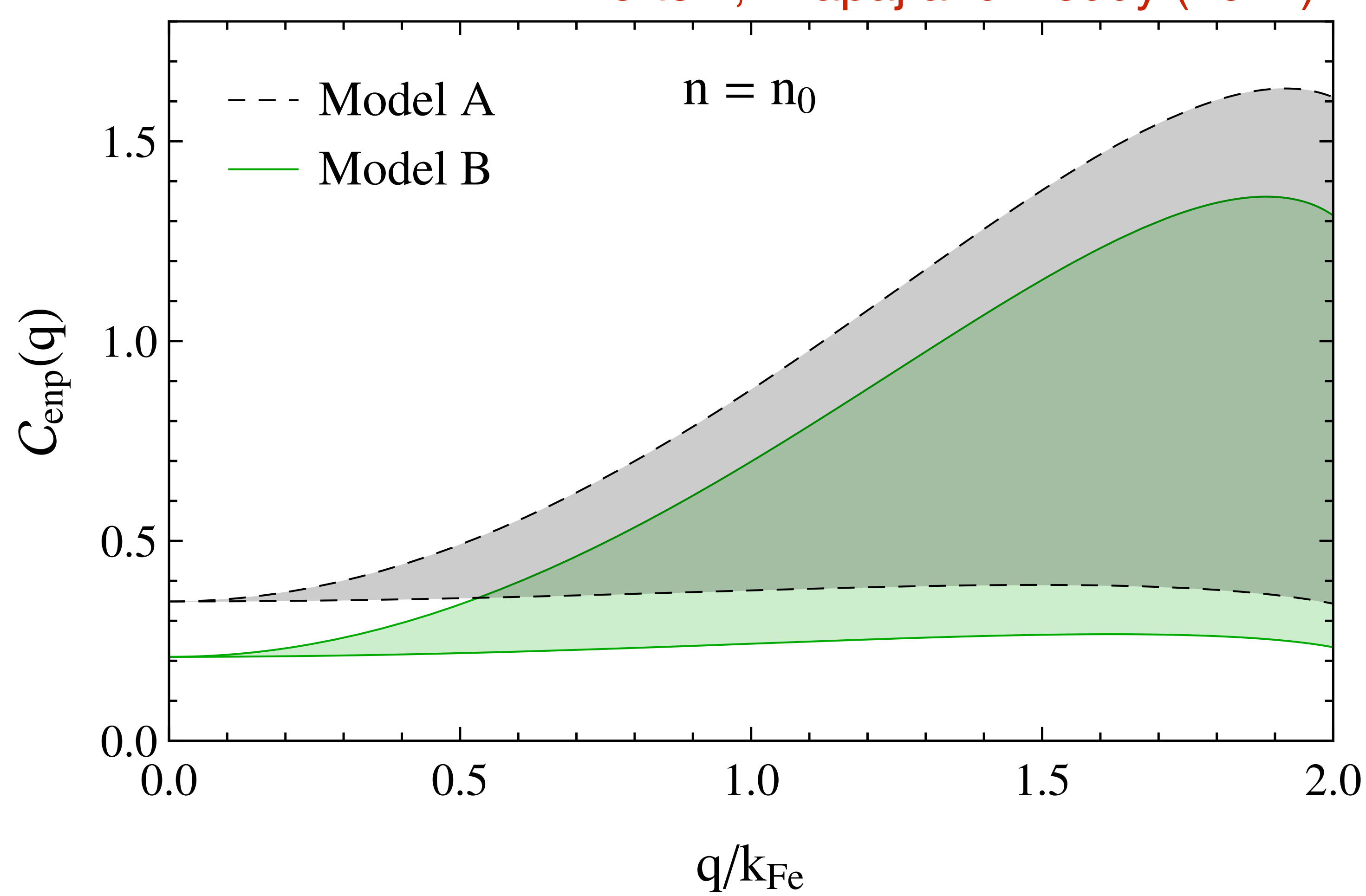
Bertoni, Rrapaj and Reddy (2014)

$$\mathcal{L}_{\gamma-\mathrm{n}} = -\sqrt{4\pi\alpha}\, V_{\mathrm{np}}\ \bar{n}\gamma_\mu n\ \Pi_{\mathrm{p}}^{\mu\nu} A_\nu$$

$$\mathcal{L}_{\mathrm{e-n}} = -\bar{e}\gamma_0 e\, \mathcal{U}_{\mathrm{enp}}(\omega, q)\ \bar{n}\gamma_0 n$$

$$\mathcal{U}_{\mathrm{enp}}(\omega, q) = \frac{-4\pi\alpha\, \mathcal{C}_{\mathrm{enp}}(\omega, q)}{q^2 + q_{\mathrm{TF}}^2}$$

$$\mathcal{C}_{\mathrm{enp}}(\omega, q) = V_{\mathrm{np}}(q)\chi_p(\omega, q)$$



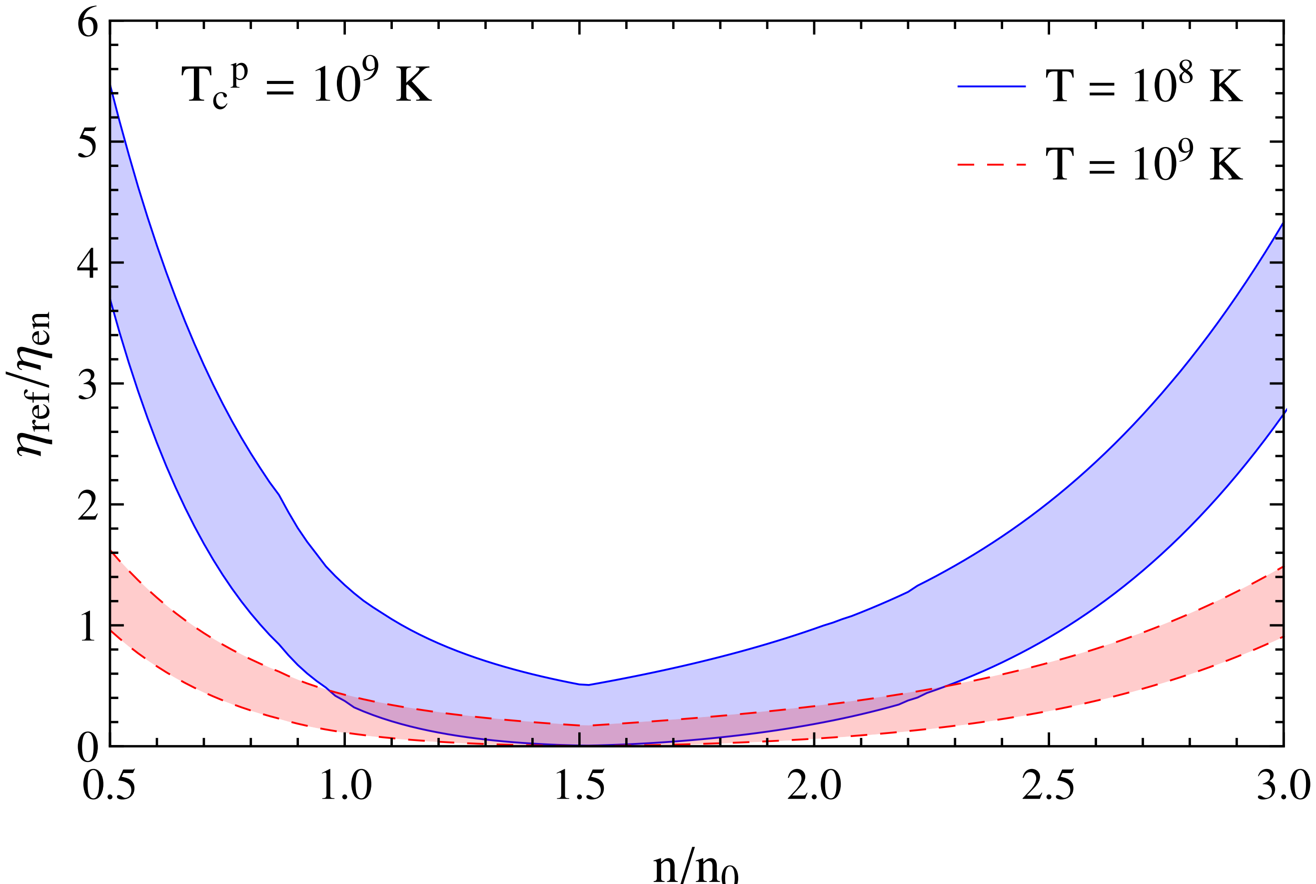
$$\chi_p(\omega, q) = \mathcal{R}e\ \Pi_p^L(\omega, q) = \mathcal{R}e \int dt\ e^{i\omega t} \int d\mathbf{r}\ e^{-i\mathbf{q}\cdot\mathbf{r}}\ \langle n_p(\mathbf{r}, t) n_p(0, 0)\rangle$$

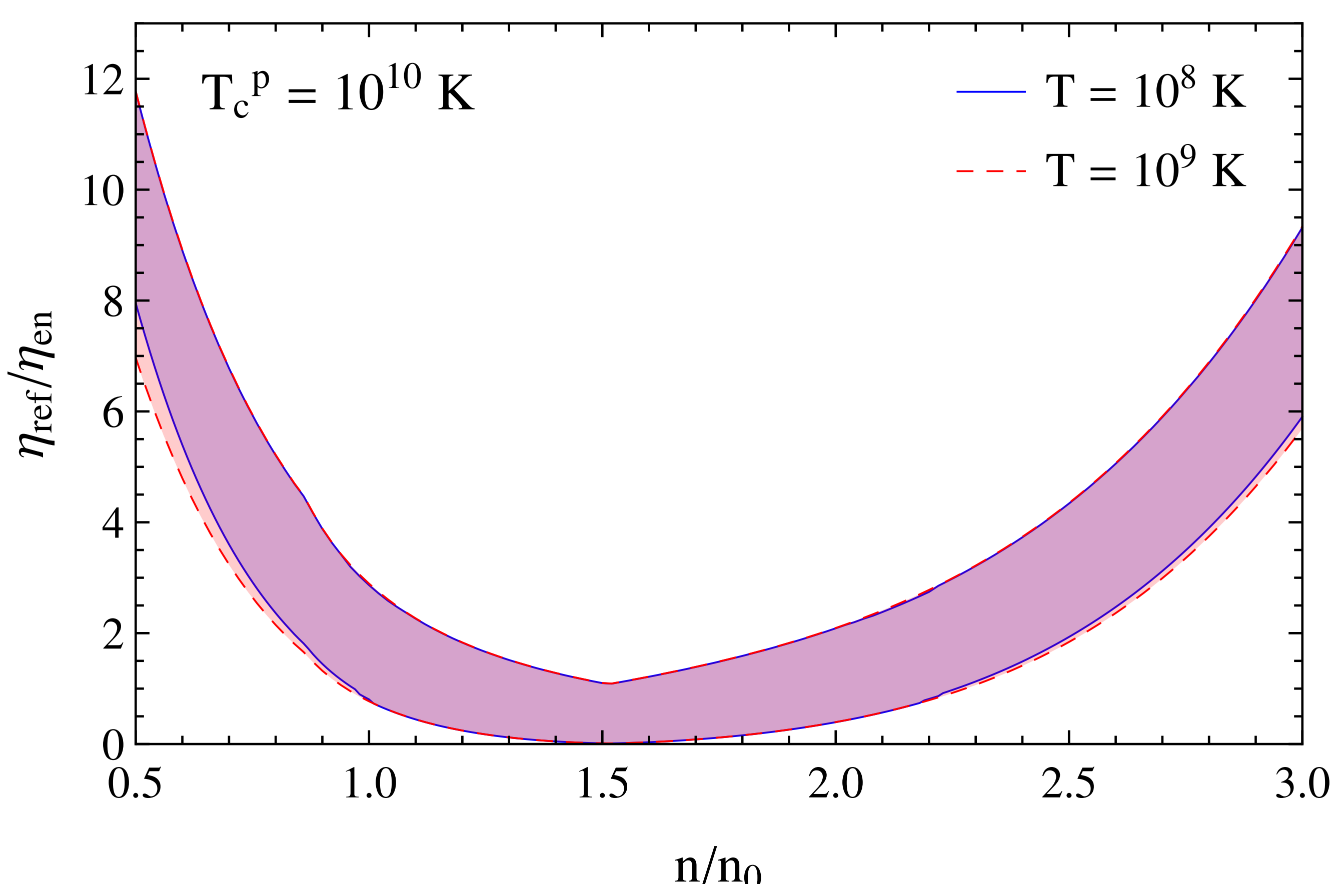
Shear Viscosity in the Core

When neutrons are normal and protons are superconducting electron-neutron scattering dominates

$$\eta_{\rm en} = \frac{k_{\rm Fe}^4}{15\pi^2}\ \langle \lambda_{en} \rangle_\eta$$

$$\eta_{\rm total} = \left(\frac{1}{\eta_{ee}} + \frac{1}{\eta_{ep}} + \frac{1}{\eta_{en}} \right)^{-1} \qquad \eta_{\rm ref} = \left(\frac{1}{\eta_{ee}} + \frac{1}{\eta_{ep}} \right)^{-1}$$





Bertoni, Rrapaj and Reddy (2014)

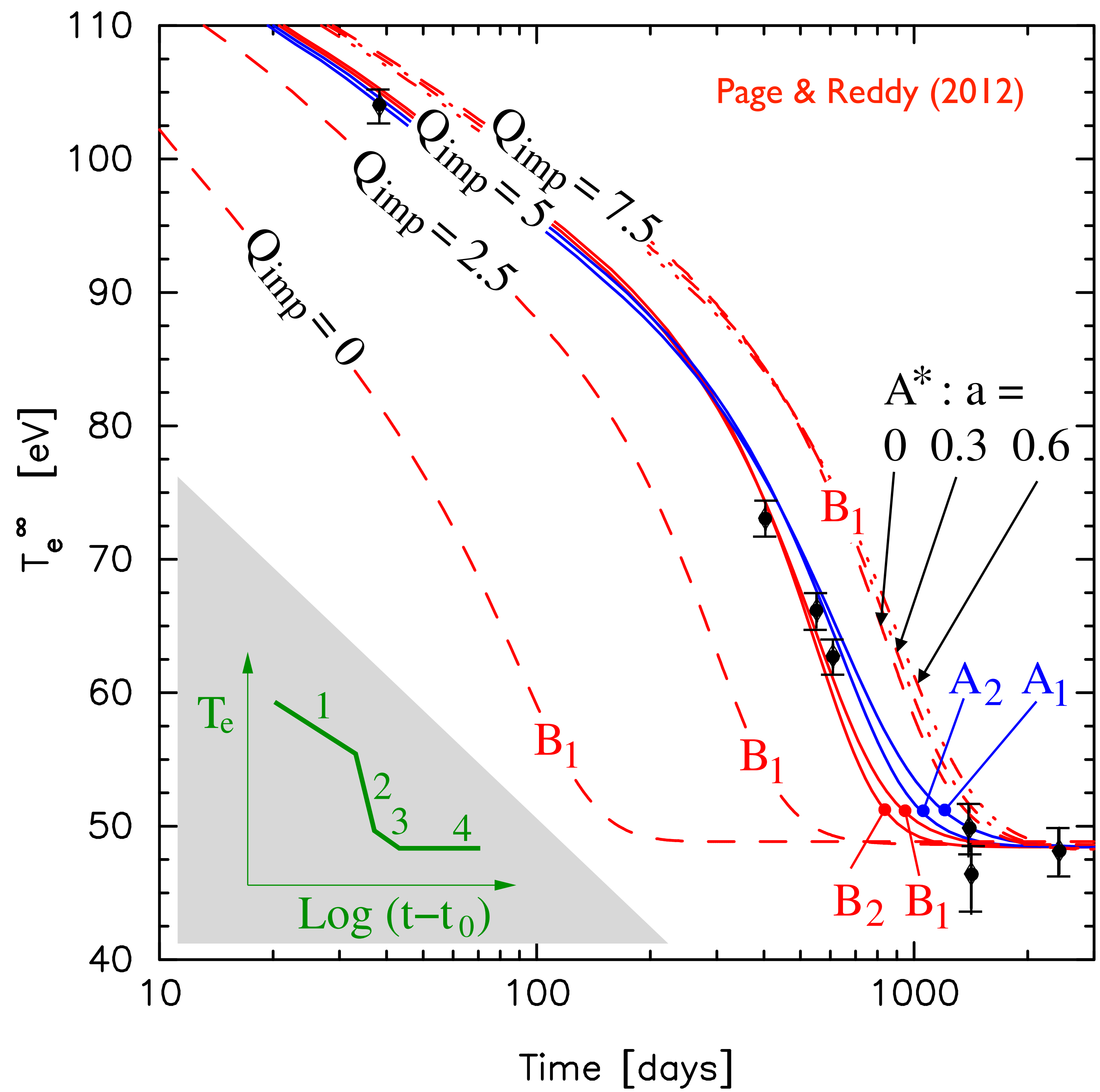
Summary

- Thermal relaxation in neutron stars is sensitive to the low temperature properties of the crust.
- Thermal and transport properties of the inner crust (super-solid) can be calculated in terms of a few low-energy constants (LEC) of a effective theory for phonons and electrons.
- Goldstone bosons in the crust and the core can decay into electron-hole states - this limits their contribution to transport.
- The induced interaction between electrons and neutrons is relevant in the neutron star core.

Unraveling thermal relaxation

- Late time signal is sensitive to inner crust thermal and transport properties.
- Impurity parameter can be fixed at earlier times. Shternin & Yakovlev (2007) Brown & Cumming (2009)
- Variations in the pairing gap (changes the fraction of normal neutrons) are discernible !

Page & Reddy (2012)



Unraveling thermal relaxation

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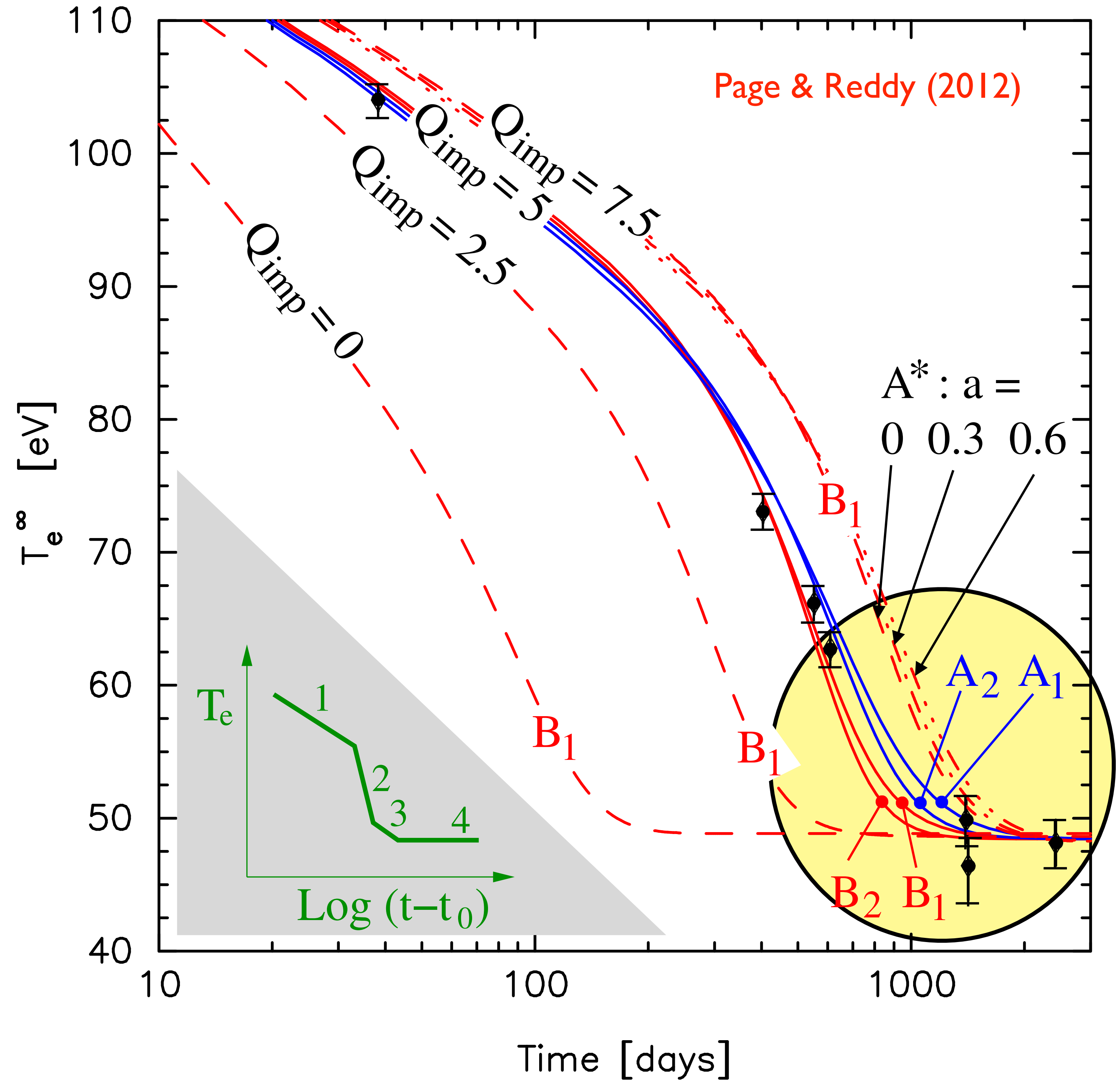

A: Low T_c - large normal fraction

B: High T_c- small normal fraction

Field theoretic analysis

Partition function in collective variables:

$$Z[A^n_\mu, A^p_\mu, g_{\mu\nu}] = \int [d\Psi_n][d\Psi_p] e^{i\mathcal{S}[\Psi_n,\Psi_I,A^n_\mu,A^p_\mu,g_{\mu\nu}]} \rightarrow \int [d\phi][d\xi^a] e^{i\mathcal{S}_{\rm eff}[\phi,\xi^a,A^n_\mu,A^p_\mu,g_{\mu\nu}]}$$

Effective Action and Lagrange Density:

$$\begin{aligned}\mathcal{S}_{\rm eff}[\phi,\xi^a,A^n_\mu,A^p_\mu,g_{\mu\nu}] &= \int d^4x\sqrt{-g}\Big[\mathcal{L}_0(\partial_\mu\phi,\partial_\mu z^a,A^n_\mu,A^p_\mu,g_{\mu\nu}) \\ &+ \mathcal{L}_1(D_\nu\partial_\mu\phi, D_\nu\partial_\mu z^a, D_\mu A^n_\nu ...) + ...\Big] .\end{aligned}$$

Partition function at constant external fields:

$$Z[\bar{A}^n_\mu, \bar{A}^p_\mu, \bar{g}_{\mu\nu}] = e^{iW[\bar{A}^n_\mu,\bar{A}^p_\mu,\bar{g}_{\mu\nu}]} = e^{-iVT\Omega[\mu_n,\mu_p,\bar{g}_{\mu\nu}]} = e^{iVT\mathcal{L}_0(0,\delta^a_\mu,\bar{A}^n_\mu,\bar{A}^p_\mu,\bar{g}_{\mu\nu})}$$

$$-\Omega[\mu_n,\mu_p,\bar{g}_{\mu\nu}] = f(X=X_0, W^a=0, H^{ab}=\bar{g}^{ab}) + \frac{1}{\sqrt{-\bar{g}}} C_1(\mu_p+m_p)$$

$$\begin{aligned} X &= g^{\mu\nu} D_\mu\phi D_\nu\phi \\ W^a &= g^{\mu\nu} D_\mu\phi\partial_\nu z^a \\ H^{ab} &= g^{\mu\nu}\partial_\mu z^a\partial_\nu z^b \end{aligned}$$

“Thermodynamic Lagrangian” $\mathcal{L}_0(X_0) = P(\sqrt{\bar{A}_\mu \bar{A}^\mu} - m_n) = P(\sqrt{X_0} - m_n = \mu_n)$

$$x^\mu \to x'^\mu = x^\mu + a^\mu(x)$$

$$g^{\mu\nu}(x) \to g'^{\mu\nu}(x') = g^{\rho\sigma}(x) \frac{\partial x'^\mu}{\partial x^\rho} \frac{\partial x'^\nu}{\partial x^\sigma}$$

$$\Psi_n(x) \to \Psi'_n(x) = \exp(-i\theta_n(x))\Psi_n(x)$$

$$A^n_\mu(x) \to A'^n_\mu(x) = A^n_\mu(x) - \partial_\mu \theta^n(x) \ ,$$

$$e^{iW[A^n_\mu, A^p_\mu, g_{\mu\nu}]} = \int [d\Psi_n][d\Psi_p] e^{i\mathcal{S}[\Psi_n, \Psi_p, A^n_\mu, A^p_\mu, g_{\mu\nu}]}$$

$$= Z[A^n_\mu, A^p_\nu, g_{\mu\nu}] \, ;$$

$$e^{iW[\bar{A}^n_\mu, \eta_{\mu\nu}]} = e^{i\mathcal{S}_{\rm eff}|_{\phi_0=0} + W_{1-\rm loop} + \cdots}$$

$$e^{iW_{1-\rm loop}} = \int [d\varphi] e^{i(\frac{1}{2}\int d^4x d^4x' \varphi(x)\varphi(x') \frac{\delta^2 \mathcal{S}_{\rm eff}}{\delta\phi(x)\delta\phi(x')}|_{\phi_0} + \ldots)}$$

$$W[A^n_\mu] = \int d^4x \mathcal{L}_{\rm eff}\big((D_\mu \phi_0[A^n_\mu]), \eta_{\mu\nu}\big) + W_{1-\rm loop}(A^n_\mu) + \ldots$$

$$= \int d^4x \left[\mathcal{L}_0\big(X_0\big) + \mathcal{L}_2[A^n_\mu] + \mathcal{L}_4[A^n_\mu]\right] + W_{1-\rm loop}(A^n_\mu) + \ldots$$

$$W[\bar{A}^n_\mu] = \int d^4x \, \mathcal{L}_0(\bar{A}^n_\mu \bar{A}^{\mu\,n}) = VT \, \mathcal{L}_0(\bar{A}^n_\mu \bar{A}^{\mu\,n})$$

$$\mu_n + \partial_0 \phi - \frac{(\partial_i \phi)^2}{2m_n}$$